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D-branes in the background of NS fivebranes

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Abstract: We study the dynamics of D-branes in the near-horizon geometry of NS fivebranes. This leads to a holographically dual description of the physics of D-branes ending on and/or intersecting NS5-branes. We use it to verify some properties of such D-branes which were deduced indirectly in the past, and discuss some instabilities of non-supersymmetric brane configurations. Our construction also describes vacua of Little String Theory which are dual to open plus closed string theory in asymptotically linear dilaton spacetimes.

Keywords: D-branes, Conformal Field Models in String Theory, Brane Dynamics in Gauge Theories, AdS-CFT Correspondance.

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1. Introduction

In the last few years it was found that embedding various supersymmetric gauge theories in string theory, as the low energy worldvolume dynamics on branes, provides an efficient tool for studying many aspects of the vacuum structure and properties of BPS states in these theories. One class of constructions (reviewed in [1]) involves systems of D-branes ending on and/or intersecting NS5-branes. For applications to gauge theory one is typically interested in taking the weak coupling limit $g_s \to 0$ (as well as the low energy limit). In this limit one might expect the system to be amenable to a perturbative worldsheet treatment, but the presence of the NS5-branes and branes ending on branes complicate the analysis.

In the absence of a derivation of the properties of D-branes interacting with NS5-branes from first principles, some of their low energy properties were postulated in the past based on symmetry considerations and consistency conditions. One of the main purposes of this paper is to derive some of these properties by a direct worldsheet study of D-branes in the vicinity of NS5-branes.

In the analysis, we will use the improved understanding of the dynamics of NS5-branes achieved in the last few years. It is now believed that in the weak coupling limit (but not necessarily at low energies) fivebranes decouple from gravity and other bulk string modes and give rise to a rich non-gravitational theory, Little String Theory (LST) [2]. In [3] it was proposed to study LST using holography. The theory on a stack of NS5-branes was argued to be holographically dual to string theory in the near-horizon geometry of the fivebranes. Many properties of LST can be understood by performing computations in string theory in this geometry. In particular, we will find below that it is an efficient way to study properties of D-branes ending on or intersecting NS5-branes.

From the point of view of LST, D-branes in the vicinity of NS5-branes give rise to new particle and extended object states in the theory, as well as new vacua, typically with reduced supersymmetry (when the branes are space-filling). Another motivation of this work is to understand the non-perturbative spectrum and dynamics of extended objects in LST, and more generally study the interplay between the non-trivial dynamics on fivebranes and the worldvolume physics of D-branes in their vicinity.

The plan of the paper is the following. In section 2 we briefly review some facts regarding the relevant brane configurations and, in particular, describe the conjectured properties of these configurations that we will try to verify. Section 3 is a review of the near-horizon geometry of NS5-branes (the CHS geometry [4]) and its holographic relation to LST. We also describe a modification of this geometry corresponding to fivebranes positioned at equal distances around a circle, which plays a role in the analysis. In section 4 we review some facts about D-branes in flat space and on a three-sphere (or SU(2) WZW model). In section 5 we study D-branes in the CHS geometry as well as its regularized version. We verify some of the properties described in section 2, and describe additional features which follow from our analysis. Some of the technical details are presented in the appendices.

2. Some properties of brane configurations

One class of brane constructions, that gave rise to many insights into gauge dynamics, involves D-branes suspended between NS5-branes. We start with a brief description of a particular example of such a construction, which realizes four-dimensional $N = 1$ supersymmetric gauge theory with gauge group $G = \mathrm{U}(N_c)$ and N_f chiral

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superfields in the fundamental representation (more precisely, N_F fundamentals Q^i, $i = 1, \ldots, N_F$, and N_F anti-fundamentals $\tilde{Q}_i$). This will help introduce the issues that will be discussed later.[1]

We will consider brane configurations in type-IIA string theory consisting of NS5-branes, D4- and D6-branes, oriented as follows:[2]

$$\begin{aligned} \text{NS5} &: \left(x^0, x^1, x^2, x^3, x^4, x^5\right), \\ \text{NS5}' &: \left(x^0, x^1, x^2, x^3, x^8, x^9\right), \\ \text{D4} &: \left(x^0, x^1, x^2, x^3, x^6\right), \\ \text{D6} &: \left(x^0, x^1, x^2, x^3, x^7, x^8, x^9\right), \\ \text{D6}' &: \left(x^0, x^1, x^2, x^3, x^4, x^5, x^7\right). \end{aligned} \tag{2.1}$$

One can check that any combination of two or more of these branes preserves four or eight supercharges. The Lorentz symmetry is broken in the presence of the branes to

$$\mathrm{SO}(9,1) \longrightarrow \mathrm{SO}(3,1)_{0123} \times \mathrm{SO}(2)_{45} \times \mathrm{SO}(2)_{89} \tag{2.2}$$

(assuming that all branes that are pointlike in (x^4, x^5) are placed at the same point in the $(4,5)$ plane, and similarly for (x^8, x^9)). To study situations with non-trivial $(3+1)$-dimensional physics (in (x^0, x^1, x^2, x^3)) some of the branes must be made finite.

Consider a configuration [5] of N_c D4-branes stretched between an NS5-brane and an NS5′-brane separated by a distance L along the x^6 direction (figure 1). At distances greater than L, the five-dimensional theory on the D4-branes reduces to a four-dimensional theory with $N = 1$ supersymmetry. The boundary conditions provided by the fivebranes imply that out of all the massless degrees of freedom on the fourbranes (described by $4-4$ strings), only the $4d$ $N = 1$ vector multiplets for $G = \mathrm{U}(N_c)$ survive. To decouple the gauge dynamics from the complications of string theory one takes the limit $g_s \to 0$, $L/l_s \to 0$, with the four-dimensional gauge coupling $g^2 = g_s l_s / L$ held fixed [1].

One can add matter in the fundamental representation of G in one of the two ways illustrated in figure 2 (which, as explained in [6], are related). One is to place N_f D6-branes between the fivebranes (figure 2a). The $4-6$ strings give N_f fundamentals of $\mathrm{U}(N_c)$, Q^i, $\tilde{Q}_i$, whose mass is proportional to the separation between the sixbranes and the fourbranes in (x^4, x^5). Alternatively, one can add to the configuration N_f D4-branes stretching from the NS5-brane to infinity (figure 2b). In this case, the N_f fundamentals of $\mathrm{U}(N_c)$ should arise from $4-4$ strings stretched between the two kinds of fourbranes. Their mass is the separation between the fourbranes in (x^4, x^5).

[1] For a more detailed discussion, references to the original literature and other constructions, see [1].

[2] In later sections we will discuss more general configurations, obtained by rotating some of the branes.

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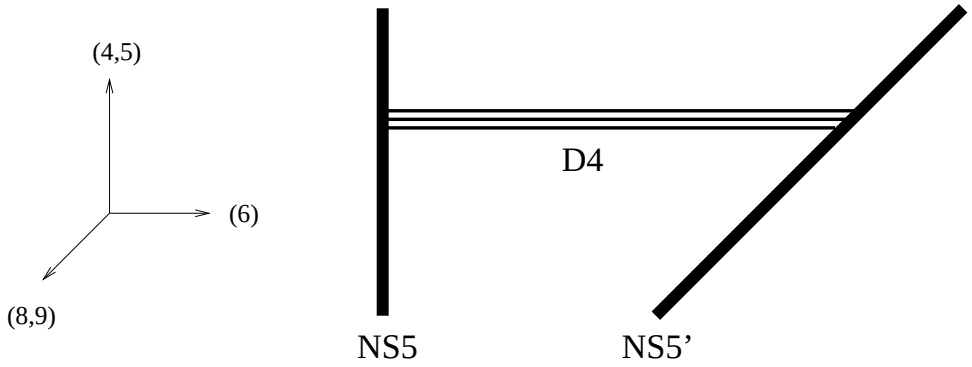

Figure 1: Brane configuration with 4D $N = 1$ U(N) SYM theory at low energy.

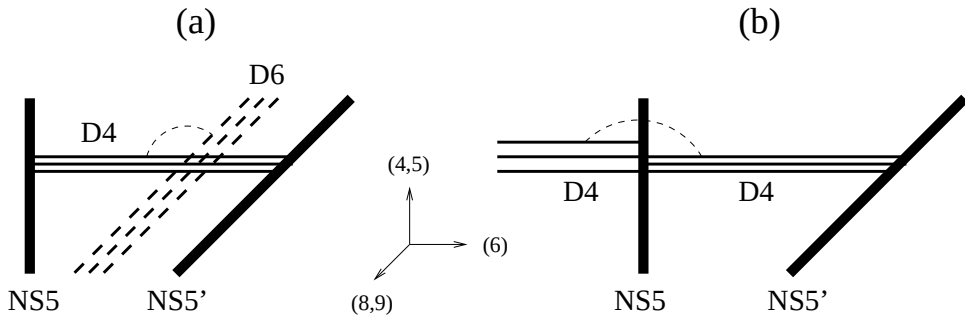

Figure 2: Adding matter to the system.

In the configuration of figure 2a, the low energy physics should be independent of the locations of the D6-branes along the interval between the fivebranes. In [7] it was pointed out that a particularly natural location for the sixbranes is at the same value of (x^4, x^5, x^6) as the NS5′-brane. As is clear from (2.1), in this case the NS5′-brane is embedded in the D6-branes; it divides them into two disconnected parts (figure 3).

Consequently, the configuration has two separate U(N_f) symmetries, acting on the two semi-infinite sixbranes, and one may attempt to interpret them as the U(N_f) × U(N_f) global symmetry of $N = 1$ SQCD, under which Q and $\tilde{Q}$ transform as $(N_F, 1)$ and $(1, \overline{N}_F)$, respectively. This would imply that $4-6$ strings connecting the D4-branes to the upper part of the D6-branes give rise at low energies to the N_f chiral multiplets in the fundamental of U(N_c), Q^i, while those that connect the fourbranes to the lower part of the sixbranes give the N_f chiral superfields in the anti-fundamental of U(N_c), $\tilde{Q}_i$, as indicated in figure 3.

Despite the fact that the two groups of sixbranes are independent, none of them can be removed from the configuration. From the string theory point of view the reason is that this would lead to violation of the RR charge that couples to D6-branes. In the low energy gauge theory the inconsistency is seen as a non-vanishing chiral

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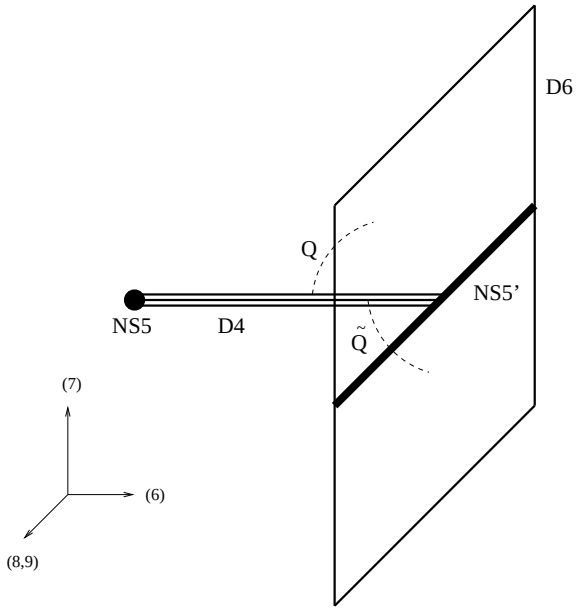

Figure 3: Brane realization of chiral symmetry.

anomaly. However, the same basic mechanism for getting chiral matter can be used to produce chiral spectra in lower dimensions [8] or in the presence of orientifolds [9, 10, 11].

The low-lying excitations of the brane configurations discussed above can be divided into two classes: those that are bound to one of the fivebranes, and those that are not. In this paper we will analyze the properties of the first class of excitations. It includes the following:

(a) $4-6$ strings connecting N_c D4-branes to N_F D6′-branes, all of which end on an NS5-brane[3] (figure $4a$). The prediction is that they give rise to a chiral spectrum: a chiral superfield Q in the (N_c, N_F) of $\mathrm{U}(N_c) \times \mathrm{U}(N_F)$.

(b) $4-4$ strings connecting fourbranes ending on an NS5-brane from opposite sides (figure $4b$). They should give rise to chiral superfields, Q in the (N_c, N_F) and $\tilde{Q}$ in the $(\overline{N}_c, \overline{N}_F)$, or hypermultiplets $(Q, \tilde{Q})$.

(c) $4-6$ strings connecting D4-branes ending on NS5-branes to D6-branes intersecting the fivebranes (figure $4c$). They should also give rise to hypermultiplets $(Q, \tilde{Q})$.

We will verify the predictions (a), (b), (c) below. Excitations which belong to the second class and which we will not analyze include the $4-4$ strings that give rise to the $N = 1$ vector superfield in figure 1, and the $4-6$ strings that give rise to

[3]The configuration of figure $4a$ can be obtained from that of figure 3 by exchanging $(x^4, x^5) \leftrightarrow (x^8, x^9)$ and removing some branes.

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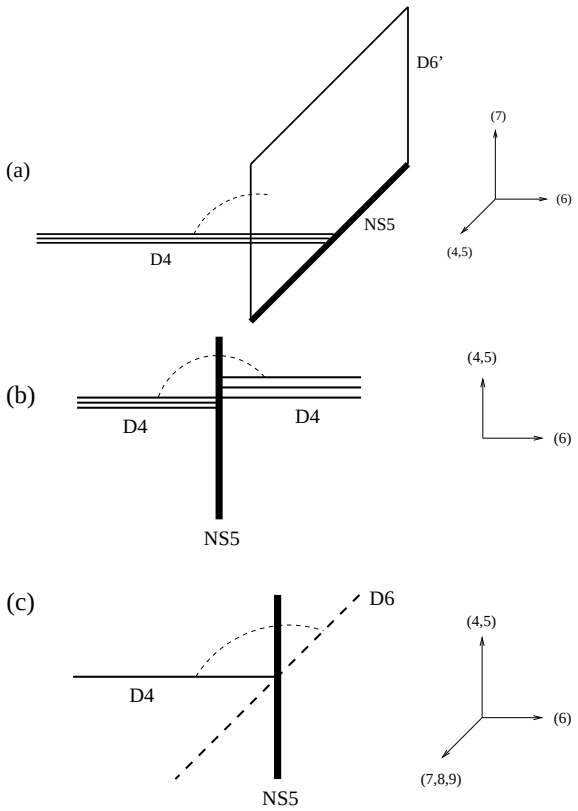

Figure 4: Open strings bounded to NS5-brane: (a) 4-6' strings, (b) 4-4 strings, (c) 4-6 strings

the fundamental chiral superfields Q^i, $\tilde{Q}_i$ in figure $2a$. The former are complicated to study, since their wavefunction is spread throughout the interval between the fivebranes and is influenced by them only via the boundary conditions they provide. The latter can be studied using standard D-brane techniques, at least when the intersection of the D4- and D6-branes is far enough from the edges of the fourbrane (see section 4). When that intersection lies on the NS5-brane (figure $4c$), one can study it using the techniques of this paper, and we will discuss this case below.

3. The near-horizon geometry of NS5-branes and holography

The background fields around a stack of k parallel NS5-branes are [4]:

$$\begin{aligned} e^{2(\Phi-\Phi_0)} &= 1+\sum_{j=1}^{k}\frac{l_s^2}{|\vec{x}-\vec{x}_j|^2}\,, \\ G_{IJ} &= e^{2(\Phi-\Phi_0)}\delta_{IJ}\,, \\ G_{\mu\nu} &= \eta_{\mu\nu}\,, \\ H_{IJK} &= -\epsilon_{IJKL}\partial^L\Phi\,. \end{aligned} \tag{3.1}$$

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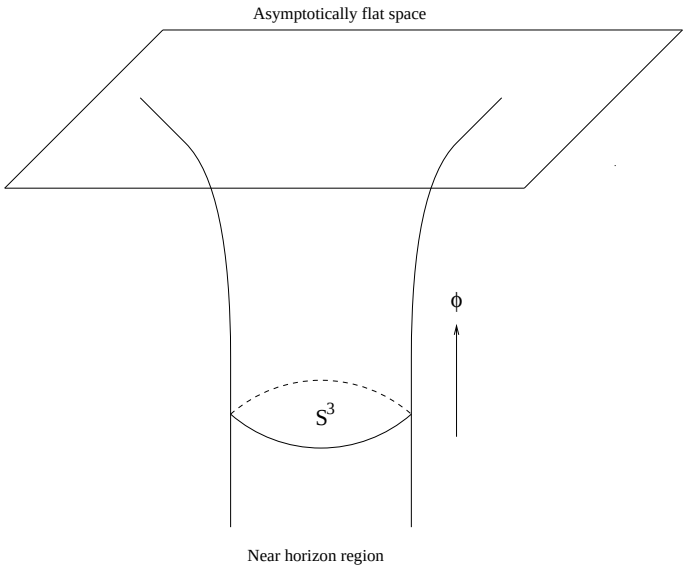

Figure 5: Background geometry around a stack of NS5-branes

$I, J, K, L = 6, 7, 8, 9$ label the directions transverse to the fivebranes (see (2.1)). $\mu, \nu = 0, 1, \ldots, 5$ are the directions along the brane. $\{\vec{x}_j\}$ are the locations of the fivebranes in $\vec{x} = (x^6, \ldots, x^9)$. H is the field strength of the NS-NS B-field; G, Φ are the metric and dilaton, respectively.

The background (3.1) interpolates between flat ten-dimensional spacetime far from the fivebranes, and a near-horizon region, in which the 1 on the right-hand side of the first line of (3.1) can be neglected (figure 5). This near-horizon region is an asymptotically linear dilaton solution. e.g. if the fivebranes are coincident, $\vec{x}_j = 0$, the near-horizon solution is

$$\begin{aligned} e^{2(\Phi-\Phi_0)} &= \frac{kl_s^2}{|\vec{x}|^2}\,, \\ G_{IJ} &= e^{2(\Phi-\Phi_0)}\delta_{IJ}\,, \\ G_{\mu\nu} &= \eta_{\mu\nu}\,, \\ H_{IJK} &= -\epsilon_{IJKM}\partial^M\Phi\,. \end{aligned} \tag{3.2}$$

String propagation in the near-horizon geometry (3.2) can be described by an exact worldsheet Conformal Field Theory (CFT) [4]. The target space is

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times \mathrm{SU}(2)\,. \tag{3.3}$$

$\mathbb{R}_\phi$ corresponds to the radial direction $r = |\vec{x}|$:

$$\phi = \frac{1}{Q}\log\frac{|\vec{x}|^2}{kl_s^2}\,, \qquad \Phi = -\frac{Q}{2}\phi\,, \tag{3.4}$$

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where we set $\Phi_0 = 0$ by rescaling $\vec{x}$. Q is related to the number of fivebranes via

$$Q = \sqrt{\frac{2}{k}}\,. \tag{3.5}$$

The CFT describing the three-sphere at constant $|\vec{x}|$ in (3.2) is an SU(2) WZW model at level k. The SU(2) group element g is related to the coordinates on the three-sphere via (e.g.):

$$g(\vec{x}) = \frac{1}{|\vec{x}|}\left[-x^6 1 + i\left(x^8\sigma_1 + x^9\sigma_2 + x^7\sigma_3\right)\right]. \tag{3.6}$$

The $\mathrm{SO}(4) \sim \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ global symmetry corresponding to rotations in the $\mathbb{R}^4$ labeled by (x^6, x^7, x^8, x^9) acts on g as $g \to h_L g h_R$, where $h_{L(R)} \in \mathrm{SU}(2)_{L(R)}$. Denoting the generators of $\mathrm{SU}(2)_L$ ($\mathrm{SU}(2)_R$) by J^a ($\bar{J}^a$), one finds that $J^3 - \bar{J}^3$ generates rotations in the (x^6, x^7) plane, while $J^3 + \bar{J}^3$ is the generator of rotations in (x^8, x^9).

Since we are dealing with the superstring, we are interested in the $N = 1$ superconformal σ-model on (3.3). Thus, in addition to the bosonic coordinates (x^μ, ϕ, g), there are worldsheet fermions $(\psi^\mu, \chi^r, \chi^a)$, $a = 1, 2, 3$, which are free (after a certain chiral rotation). The worldsheet $N = 1$ superconformal generators are:

$$\begin{aligned} T(z) &= -\frac{1}{2}(\partial x^\mu)^2 - \frac{1}{2}\psi^\mu\partial\psi_\mu - \frac{1}{2}(\partial\phi)^2 - \frac{Q}{2}\partial^2\phi - \frac{1}{2}\chi^r\partial\chi^r - \frac{1}{k}J^aJ^a - \frac{1}{2}\chi^a\partial\chi^a\,, \\ G(z) &= i\psi_\mu\partial x^\mu + i\chi_r\partial\phi + iQ\left(\chi_a J^a - i\chi_1\chi_2\chi_3 + \partial\chi_r\right). \end{aligned} \tag{3.7}$$

Here J^a are the bosonic SU(2) currents of level $k_B \equiv k - 2$. The total SU(2) current algebra of level k is generated by the currents $J^a_{\mathrm{total}} = J^a + J^a_F$, where $J^a_F = -(i/2)\epsilon^{abc}\chi_b\chi_c$ is the contribution of the fermions. Note, in particular, that this construction only makes sense for $k_B \geq 0$, i.e. for two or more fivebranes. One also imposes a chiral GSO projection $(-)^{F_L} = (-)^{F_R} = 1$. The GSO projected theory on the background (3.1) and (3.3) preserves sixteen supercharges — the NS5-brane is a half BPS object.

An interesting feature of the near-horizon geometry (3.2) is that in the vicinity of the fivebranes, $|\vec{x}| \to 0$, an infinite "throat" appears (for two or more fivebranes), corresponding to $\mathbb{R}_\phi$ in (3.3). In [3] it was proposed to interpret it in terms of holography. String theory in the background (3.3) was conjectured to be equivalent to the theory on the fivebranes (LST).

The map between the "bulk" ($10d$) and "boundary" ($6d$) theories is the following. On-shell observables in the bulk theory, such as vertex operators in the background (3.3) which correspond to non-normalizable wavefunctions supported at the "boundary", $\phi \to \infty$ on $\mathbb{R}_\phi$, are mapped to off-shell observables in the $6d$ fivebrane theory. Non-normalizable vertex operators on (3.3) depend among other things on the six-dimensional momentum k^μ, which is interpreted as off-shell momentum in the $6d$ LST.

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Normalizable eigenstates of the hamiltonian on (3.3) correspond to on-shell states in LST. One way to compute the spectrum of these states is to study correlation functions of non-normalizable vertex operators and look for singularities as a function of k_μ. Poles at $k_\mu^2 = -M_i^2$ signal the presence of normalizable states with that mass in the spectrum of the theory.

The worldsheet theory (3.2)–(3.7) is singular. The string coupling $g_s \sim \exp(-Q\phi/2)$ diverges as $\phi \to -\infty$ (i.e. as one approaches the fivebranes (3.4)). Therefore, the weakly coupled ten-dimensional description is only useful for studying those aspects of LST that can be analyzed at large positive ϕ. This includes identifying a large set of observables (such as the aforementioned non-normalizable vertex operators), and their transformation properties under the symmetries of the theory (e.g. $6d$ super-Poincare symmetry). Normalizable states are difficult to analyze, since their wave-functions tend to be supported in the strongly coupled region $\phi \to -\infty$. Equivalently, correlation functions of non-normalizable operators in linear dilaton vacua are typically not computable without specifying the cutoff at large negative ϕ.

If one is interested in studying the theory on $k > 1$ coincident fivebranes, one must face this strong coupling problem. However, to make contact with section 2 we are in fact mainly interested in the case where the fivebranes are separated. In that case one might hope that the strong coupling region will be absent, e.g. because the CHS solution with the throat only makes sense when there are at least two coincident fivebranes. Indeed, one can show that this is the case.

If, for example, we distribute the k fivebranes at equal distances around a circle in the (x^6, x^7) plane, which breaks the global symmetry

$$\mathrm{SO}(5,1) \times \mathrm{SO}(4) \longrightarrow \mathrm{SO}(5,1) \times \mathrm{SO}(2) \times Z_k \tag{3.8}$$

the background (3.3) changes as follows. Decompose $\mathrm{SU}(2) \to \mathrm{U}(1) \times \mathrm{SU}(2)/\mathrm{U}(1)$. The $\mathrm{U}(1)$ is the CSA generator J^3_{total}, which can be bosonized as

$$J^3_{\mathrm{total}} = 2i\sqrt{\frac{k}{2}}\partial Y \ , \tag{3.9}$$

where Y is a scalar field normalized such that $\langle Y(z)Y(w)\rangle = -\frac{1}{4}\log(z-w)$, a normalization that will be convenient later; it differs by a factor of four from that of [12, 13]. In the original CHS solution, (ϕ, Y) describe an infinitely long cylinder with a string coupling that varies along the cylinder, diverging at one end and going to zero at the other. Separating the fivebranes replaces it [12]–[15] with a semi-infinite cigar [16] $\mathrm{SL}(2)/\mathrm{U}(1)$ in which the strong coupling region is absent, or equivalently with $N = 2$ Liouville, in which the strong coupling region is suppressed by a superpotential which goes like $\exp[-(\hat{\phi} + i\hat{Y})/Q]$, where $(\hat{\phi}, \hat{Y})$ are the superfields whose bosonic components are (ϕ, Y). For reasons explained in [13] it is more convenient to use the coset

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(cigar) description in this case. The full background replacing (3.2) is

$$\mathbb{R}^{5,1} \times \frac{\mathrm{SL}(2,\mathbb{R})_k}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)} \,. \tag{3.10}$$

The global SO(2) symmetry in (3.8) corresponds to translations in Y (i.e. rotations of the cigar around its axis). The Z_k charge in (3.8) is the winding number around the cigar, a U(1) which is broken down to Z_k, since winding number can slip off the tip of the cigar. It should be noted that the product in (3.10) is not direct, since the GSO projection relates the different factors. In the GSO projected theory, the winding number around the cigar can be fractional ($\in Z/k$). The fractional part of the winding number is the Z_k charge mentioned above.

Since the background (3.10) can be made arbitrarily weakly coupled,[4] one can use it to compute correlation functions in LST in a weak coupling regime in its moduli space of vacua [13]. This is done by constructing BRST invariant observables on (3.10) and computing their worldsheet correlation functions, using the results of [17]. For a detailed analysis we refer the reader to [13]; here we mention a few facts that will play a role below.

Consider the (NS, NS) sector of the theory.[5] The observables are primaries of the $N = 1$ superconformal algebra (3.7) with scaling dimension $(h, \bar{h}) = (1/2, 1/2)$. The $\mathbb{R}^{5,1}$ and $\mathrm{SU}(2)/\mathrm{U}(1)$ are well known SCFT's. The former is a free field theory; the latter, an $N = 2$ minimal model with $c = 3 - (6/k)$. The $\mathrm{SL}(2)/\mathrm{U}(1)$ SCFT is $N = 2$ superconformal as well; it has central charge $c = 3 + (6/k)$. The $N = 2$ primaries $V_{j;m,\bar{m}}$ have scaling dimensions

$$(h, \bar{h}) = \frac{1}{k}\left(m^2 - j(j+1), \bar{m}^2 - j(j+1)\right) \tag{3.11}$$

and

$$(m, \bar{m}) = \frac{1}{2}(p + wk, p - wk)\,, \tag{3.12}$$

where $p, w \in \mathbb{Z}$ are momentum and winding around the cigar, respectively. As mentioned before, in the GSO projected theory (3.10) one finds

$$w \in \frac{1}{k}\mathbb{Z} \tag{3.13}$$

while the momentum p is still integer.

Unitarity and non-normalizability limit the range of j to

$$j \in \mathbb{R}\,, \quad -\frac{1}{2} < j < \frac{k-1}{2}\,. \tag{3.14}$$

[4] by decreasing the value of the string coupling at the tip of the cigar. This value is related to the radius of the circle on which the fivebranes lie [13].

[5] The other sectors can be reached by applying the spacetime supercharges [12].

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There are also delta-function normalizable operators with $j = -1/2 + is$, $s \in \mathbb{R}$. These do not give rise to off-shell observables in the theory, but rather should be thought of as producing a continuum of states above a gap in LST.[6] All other observables (in addition to $V_{j;m,\bar{m}}$) can be obtained by acting with $N = 2$ superconformal generators on these primaries.

By analyzing correlation functions of $V_{j;m,\bar{m}}$, one finds [13] that poles in correlators correspond to discrete representations[7] of SL(2)

$$|m| = j + n\,, \qquad |\bar{m}| = j + \bar{n}\,; \quad n, \bar{n} = 1, 2, 3, \ldots\,. \tag{3.15}$$

This leads to a discrete spectrum of states in LST, which exhibits Hagedorn growth at high energy.

4. Some properties of D-branes

In this section we review some properties of D-branes in flat space and on S^3, in preparation for our discussion of D-branes in the CHS geometry (3.3), and its regularized version (3.10).

4.1 $4-6$ strings in flat space

Later, we will analyze $4-6$ strings connecting D4-branes and D6′-branes, both of which end on a stack of NS5-branes (figure $4a$). We start by reviewing the simpler case of intersecting infinite D4- and D6′-branes in flat space [20, 21].

Consider an open string, one of whose ends is on a D4-brane. The other end may be either on the same brane or on another brane. We want to study the emission of a $4-6'$ string from the D4 boundary of this string. After the emission, this boundary of the emitting string will lie on a D6′-brane (figure 6).

The worldsheet of an open string is a strip, or equivalently the upper half plane. On the upper half plane, worldsheet time evolves radially. Equal time surfaces are half circles around the origin, with early times corresponding to small circles. The boundary of the upper half plane (the real line $\mathrm{Im}\, z = 0$) corresponds to the ends of the string and is divided into two parts: $z > 0$ or $\sigma = 0$ in figure 6, which lies on the D4-brane, and $z < 0$ or $\sigma = \pi$.

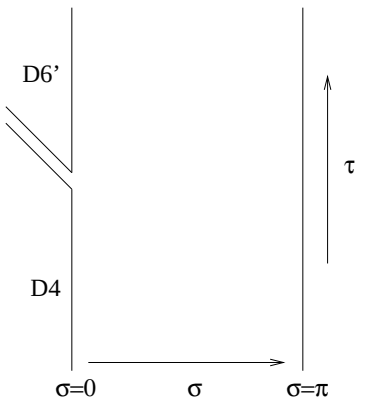

Figure 6: Emission of a 4-6' string.

The emission of an open $4-6'$ string from the boundary $\sigma = 0$ is described by an insertion of a vertex operator V at some point $z > 0$ on the boundary. The

[6] The relation between states and operators in the cigar CFT is subtle and very similar to that in Liouville theory, described in [18].

[7] In some cases one also finds poles corresponding to $m = j$ [19].

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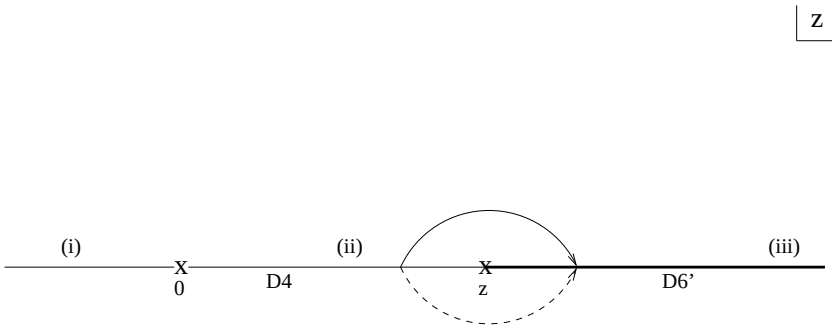

Figure 7: Upper half complex plane description of the world sheet of an open string emmiting a 4-6' string at the point z.

boundary now splits into three components: (i) $z' < 0$, the spectator boundary at $\sigma = \pi$, (ii) $0 < z' < z$, which lies on the fourbrane, and (iii) $z' > z$, which is on the sixbrane (see figure 7). The boundary conditions for, say, the coordinate x^4 are Dirichlet, $\partial x^4(z') + \overline{\partial} x^4(z') = 0$, in region (ii), and Neumann, $\partial x^4(z') - \overline{\partial} x^4(z') = 0$, in region (iii).

The vertex operator $V(z)$ changes the boundary conditions of x^4 from Neumann to Dirichlet. If one inserts the holomorphic current ∂x^4 into the worldsheet, and moves it along a small upper semicircle around z (see figure 7), from $z' < z$ to $z' > z$ (the operator $\overline{\partial} x^4$ is necessarily transported along the mirror image lower semicircle between these two points), the relative sign of the two operators has to flip. This means that the operator product $V(z)\partial x^4(z')$ has to have a square root branch cut in $z - z'$. Operators with such a cut are twist operators, familiar from orbifold CFT (see e.g. [22]). The lowest dimension operator of this type, σ, has dimension 1/16.

The same arguments apply to all the coordinates for which the $4 - 6'$ string has Dirichlet-Neumann boundary conditions; hence, the operator V contains a twist field for (x^4, x^5, x^6, x^7), σ_{4567}, with total dimension 1/4. We will be mainly interested in emission of spacetime bosons, in which case V belongs to the NS sector of the worldsheet CFT. Therefore, it has to be local with respect to $G = i\sum \psi_a \partial x^a$, the worldsheet superconformal generator. Since G has a square root branch cut with respect to $x^{4,5,6,7}$, it must also have a cut with respect to $\psi_{4,5,6,7}$. This implies that V has to contain the spin field S_{4567} for these worldsheet fermions. This operator also has dimension 1/4.

The directions $x^{0,1,2,3}$ are common to the D4- and D6′-branes, and are treated as in standard open string theory with Neumann boundary conditions. The remaining coordinates (x^8, x^9) have Dirichlet-Dirichlet boundary conditions. If the D4-brane is, say, at $x^8 = 0$, while the D6-brane is at $x^8 = a$ (and both at $x^9 = 0$), then similarly to the discussion above, a bulk operator of the form $\exp(ikx^8)$ when moved around the above mentioned semi-circle from a point on the D4 to a point on the D6′, has to pick

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a factor of $\exp(ika/2)$, together with another such contribution from the mirror path of the lower semicircle. This means that the operator product $V(z)\exp(ikx^8(z'))$ has a branch cut of the form $(z-z')^{ka/2\pi}$ and the product with the corresponding right-moving operator has a cut with the opposite sign. The appropriate boundary operator is $\exp(i(a/\pi)(x_L^8(z)-x_R^8(z)))$. This operator generates "winding number" a/π along x^8 — in agreement with the geometrical picture of a $4-6'$ string stretched a distance a along x^8. Its dimension is $a^2/2\pi^2$.

Collecting all the factors, we get the following vertex operator describing the (bosonic) ground state of a $4-6'$ string stretched between a D4-brane at $(x^8,x^9)=(0,0)$ and a D6′-brane at $(x^8,x^9)=(a,b)$,

$$V=e^{-\varphi}\sigma_{4567}S_{4567}e^{\frac{i}{\pi}(a(x_L^8-x_R^8)+b(x_L^9-x_R^9))}e^{ik_\mu x^\mu}\ , \tag{4.1}$$

where k_μ ($\mu=0,1,2,3$) is the $4d$ spacetime momentum. φ is the bosonized superconformal ghost. The vertex operator (4.1) is written in the -1 picture; one can check that the coefficient of $e^{-\varphi}$ in (4.1) is an $N=1$ superconformal primary, which is a necessary condition for its BRST invariance. The requirement that it has worldsheet dimension 1/2, which is also necessary for BRST invariance, implies that the mass squared of the ground state of the $4-6'$ string is $-k_\mu^2=\frac{1}{\pi^2}(a^2+b^2)$, as one would expect (we work in a convention $\alpha'=1/2$, in which the scalars x are canonically normalized on the boundary and the tension of the fundamental string is $T=1/\pi$).

In particular, when the D6′-brane intersects the D4-brane, i.e. when $a=b=0$, this mass vanishes. The vertex operator (4.1) describes a particle which transforms as a scalar under $(3+1)$-dimensional Lorentz rotations. The spin field S_{4567} has 4 components, half of which are projected out by the GSO projection, so (4.1) actually describes two real scalar particles. For the applications described in section 2 it is sometimes useful to consider not one but a stack of N D4-branes. In that case, the two scalars (4.1) transform in the fundamental representation ($\mathbf{N}$) of the U(N) gauge symmetry on the fourbranes.

In addition to the $4-6'$ strings described above there are also $6'-4$ strings which have similar properties but transform in the $\overline{\mathbf{N}}$ of U(N). Altogether we have two complex scalars, Q in the $\mathbf{N}$ and $\tilde{Q}$ in the $\overline{\mathbf{N}}$ of U(N). The system of D4- and D6′-branes preserves eight supercharges ($N=2$ SUSY in the four dimensions (0123)) and acting with the supercharges on (4.1) completes a hypermultiplet transforming in the fundamental representation of U(N).

4.2 D-branes on the SU(2) group manifold

We next turn to some facts regarding D-branes on a group manifold G, focusing on the case $G=\mathrm{SU}(2)$ (D-branes on group manifolds have been studied, for instance, in [23]–[39]). In the absence of D-branes, the WZW model has an affine $G_L\times G_R$ symmetry. If $g(z,\overline{z})$ is a map from the worldsheet to the group G, the symmetry

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acts on it as:

$$g \longrightarrow h_L(z) g h_R(\overline{z})\,. \tag{4.2}$$

If the worldsheet has a boundary, there is a relation between left-moving and right-moving modes, and the $G_L \times G_R$ symmetry is broken. One can still preserve some diagonal symmetry G, say the symmetry

$$g \longrightarrow hgh^{-1} \tag{4.3}$$

corresponding to $h_L = h_R^{-1} = h$ in (4.2). The presence of this symmetry constrains the boundary conditions that can be placed on g. Allowing $g(\text{boundary}) = f$ for some $f \in G$ we must also allow $g(\text{boundary}) = hfh^{-1}$ for every $h \in G$. This means that g on the boundary takes value in the conjugacy class containing f [28]. For $G = \mathrm{SU}(2)$, conjugacy classes are parametrized by a single angle θ, $0 \leq \theta \leq \pi$, corresponding to the choice $f = \exp(i\theta\sigma_3)$. Thinking of $\mathrm{SU}(2)$ as the group of three-dimensional rotations, the conjugacy class $\mathbb{C}_\theta$ is the set of all rotations of angle 2θ about any axis. A boundary condition which preserves (4.3) is then

$$g(\text{boundary}) \in \mathbb{C}_\theta\,. \tag{4.4}$$

Since h_L is generated by the currents J^a while h_R is generated by $\overline{J}^a$, the invariance of the boundary condition (4.4) under (4.3) implies that the currents satisfy [36]:

$$J^a = \overline{J}^a\,; \quad a = 1, 2, 3 \tag{4.5}$$

on the boundary. Not any value of θ in (4.4) gives rise to a consistent model [24, 28, 30]. Recall that for a general group G, the level k WZW action has the form

$$S = \int_\Sigma d^2 z L^{\mathrm{sm}} + \int_B L^{\mathrm{WZ}}\,, \tag{4.6}$$

where $L^{\mathrm{sm}} = \frac{k}{4\pi}\mathrm{Tr}(\partial g^{-1}\overline{\partial} g)$ is the sigma model part of the action and $L^{\mathrm{WZ}} = \frac{k}{4\pi}\omega^{(3)}$ is the Wess-Zumino term. Here Σ is the worldsheet Riemann surface, $\omega^{(3)} = \frac{1}{3}\mathrm{Tr}((g^{-1}dg)^3)$ is a closed three-form, and B is a three-dimensional manifold whose boundary is the worldsheet. When the worldsheet Σ has itself boundaries, it cannot be the boundary of a three-dimensional manifold, since a boundary cannot have boundary. To define the WZ term in (4.6) for this case, one should fill the holes in the worldsheet by adding discs, and extend the mapping from the worldsheet into the group manifold to these discs. One further demands that the whole disc D is mapped into a region (which we will also refer to as D) inside the conjugacy class in which the corresponding boundary lies. B will then be defined as a three-manifold bounded by the union $\Sigma \bigcup D$, which now has no boundaries.

The resulting action should preserve the symmetry (4.2) in the bulk of the worldsheet which tends to (4.3) on the boundary. Take $\mathbb{C}$, the conjugacy class containing D, to be the class of a fixed group element f, i.e.

$$C = \left\{hfh^{-1} | h \in G\right\}. \tag{4.7}$$

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Let $\delta_L g$ be an infinitesimal variation of $g(z,\bar{z})$ in the bulk such that $\overline{\partial}(\delta_L g g^{-1}) = 0$ and $\delta_R g$ a variation for which $\partial(g^{-1}\delta_R g) = 0$. On the boundary of Σ, where $g \in \mathbb{C}$ is parametrized as $g = hfh^{-1}$, $\delta_L g g^{-1} = -g^{-1}\delta_R g = \delta h h^{-1}$. By the above symmetry the variation $(\delta_L + \delta_R)S$ should vanish. In the interior of the disc we should allow an arbitrary variation δh since the location of the auxiliary disc inside $\mathbb{C}$ has no physical significance and cannot influence the action. Using the identity

$$\delta\omega^{(3)} = d\,\mathrm{Tr}\left(\left(g^{-1}\delta g\right)\left(g^{-1}dg\right)^2\right) \tag{4.8}$$

one gets

$$(\delta_L + \delta_R)S = \frac{k}{4\pi}\left[\int_D \mathrm{Tr}\left(\left(g^{-1}\delta g\right)\left(g^{-1}dg\right)^2\right) + \int_{\partial\Sigma} A\right], \tag{4.9}$$

where A is the one-form

$$A = \mathrm{Tr}\left[\left(h^{-1}\delta h\right)\left(f^{-1}h^{-1}dhf - fh^{-1}dhf^{-1}\right)\right]. \tag{4.10}$$

The first term in (4.9) comes from the change in the region of integration of L^{WZ} resulting from the variation of D. The second term is a boundary correction to the symmetry (4.2). Clearly the right-hand side of (4.9) is not zero for a non-trivial mapping of $\partial\Sigma$ and of D into $\mathbb{C}$.

To fix the symmetry we have to modify the action by adding to it an integral over the disc D of some two form $\omega^{(2)}$, defined on $\mathbb{C}$, such that its variation will cancel (4.9). The proper action has now the form

$$S = \int_\Sigma d^2z L^{\mathrm{sm}} + \int_B L^{\mathrm{WZ}} + \frac{k}{4\pi}\int_D \omega^{(2)}. \tag{4.11}$$

The variation of this action is

$$(\delta_L + \delta_R)S = \frac{k}{4\pi}\left[\int_D \mathrm{Tr}\left(\left(g^{-1}\delta g\right)\left(g^{-1}dg\right)^2\right) - \int_{\partial D} A + \int_D \delta\omega^{(2)}\right], \tag{4.12}$$

where we used the fact that $\partial\Sigma$ and ∂D are identical curves with opposite orientations. The vanishing of (4.12) fixes $\delta\omega^{(2)}$ as

$$\delta\omega^{(2)} = dA - \mathrm{Tr}\left(\left(g^{-1}\delta g\right)\left(g^{-1}dg\right)^2\right). \tag{4.13}$$

Expressing the r.h.s. of (4.13) in terms of the group element h parametrizing C as in (4.7) one gets $\delta\omega^{(2)}$ explicitly on $\mathbb{C}$. Its solution is [28, 30]

$$\omega^{(2)} = \mathrm{Tr}\left[\left(h^{-1}dh\right)f^{-1}\left(h^{-1}dh\right)f\right]. \tag{4.14}$$

Notice that (as implied by (4.13)) on the conjugacy class C, $d\omega^{(2)} = -\omega^{(3)}$.

The modified action (4.11) is independent, by construction, of continuous deformations of D inside $\mathbb{C}$. However, in general, the second homotopy of a conjugacy class is non trivial. If we compare then the value of the action for D and D', two

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different choices of embedding the disc in C with the same boundary, D' may not be a continuous deformation of D in $\mathbb{C}$. In that case the above analysis does not imply that the two ways to evaluate the action (4.11) agree. Since there is no natural way to choose between the two embeddings, (4.11) is not yet a well defined action. In particular, for $G = \mathrm{SU}(2)$ the conjugacy classes C have the topology of S^2, the two-sphere generated by all possible axes of rotation by a fixed angle in three dimensions. One may then choose D and D' such that their union covers the whole of S^2. In that case the difference between the action S_D, the value of (4.11) with embedding D, and $S_{D'}$ with embedding D' is

$$\Delta S = \frac{k}{4\pi}\left[\int_B \omega^{(3)} + \int_C \omega^{(2)}\right], \tag{4.15}$$

where B is the three-volume in $\mathrm{SU}(2)$ bounded by the two-sphere C (figure 8).

For the case of $\mathrm{SU}(2)$, which has the topology of S^3, the form $\omega^{(3)}$ is $\omega^{(3)} = 4\Omega^{(3)}$ where $\Omega^{(3)}$ is the volume form on the unit three-sphere. For C in (4.7) corresponding to $f = \exp(i\theta\sigma_3)$, the first term in (4.15) is

$$\int_B \omega^{(3)} = 8\pi\left(\theta - \frac{1}{2}\sin(2\theta)\right). \tag{4.16}$$

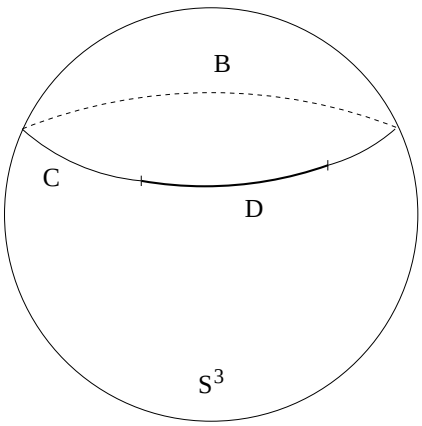

Figure 8: Region of the integrations in (4.15) on the group manifold.

As to the two-form $\omega^{(2)}$, from eq. (4.14) we see that $\omega^{(2)}[h] = \omega^{(2)}[qh]$ for any fixed element $q \in \mathrm{SU}(2)$. This implies that this form is proportional to $\Omega^{(2)}$, the volume form of the unit two-sphere. The expression (4.14) for h near the identity gives then for the conjugacy class $\mathbb{C}_\theta$,

$$\omega^{(2)} = \sin(2\theta)\Omega^{(2)}\,. \tag{4.17}$$

This gives for the change in the action for two topologically different embeddings in (4.15)

$$\Delta S = 2k\theta\,. \tag{4.18}$$

Although this is non zero, the quantum theory is still well defined if ΔS is an integral multiple of 2π. We find then that the possible conjugacy classes on which a boundary state can live are quantized, the corresponding θ must satisfy

$$\theta = 2\pi\frac{j}{k} \tag{4.19}$$

with j integer or half integer satisfying $0 \le j \le k/2$.

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Thus, there are $k+1$ different boundary conditions preserving the symmetry (4.3) which one can impose on the $SU(2)_k$ group manifold. This matches nicely with the algebraic analysis of Cardy [40], who found that in a general rational CFT on a worldsheet with boundary, to each primary field of the chiral algebra there corresponds a boundary state preserving the diagonal chiral algebra, the analog of (4.3). In the case of the SU(2) WZW model, the chiral algebra is affine $SU(2)_k$, and there are $k+1$ different primary fields corresponding to representations of (integer or half integer) spin $0 \leq j \leq k/2$. It is natural, [28, 30], to identify each of the $k+1$ different geometric boundary conditions corresponding to j in (4.19) with the algebraic boundary state corresponding to the same j.

Since all of these boundary states preserve (4.3), the open strings stretching between them should have well-defined transformation properties under the diagonal chiral algebra. According to [40], the open strings stretched between a boundary state corresponding to a primary field j and another boundary state corresponding to primary field j', belong to the representations of the chiral algebra which appear in the fusion of j and j'.

We have chosen the boundary conditions (4.4) such that the particular diagonal subgroup G of $G \times G$ defined in eq. (4.3) will survive them. This is of course not a unique choice. One can act on these boundary conditions with any element of the $G \times G$ symmetry group to get equivalent boundary conditions which preserve a different diagonal subgroup. Thus we can multiply the conjugacy class $\mathbb{C}_\theta$ in eq. (4.4) from the right (which is the same as multiplying from the left) by any group element f to get modified boundary conditions

$$g(\text{boundary}) \in \mathbb{C}_\theta f \,. \tag{4.20}$$

These boundary conditions also preserve a diagonal subgroup, since the set $\mathbb{C}_\theta f$ satisfies

$$\mathbb{C}_\theta f = h(\mathbb{C}_\theta f) f^{-1} h^{-1} f \tag{4.21}$$

for any $h \in G$. Therefore, the boundary conditions (4.20) preserve the diagonal subgroup of $G_L \times G_R$ defined by $h_R = f^{-1} h_L^{-1} f$. In terms of the infinitesimal generators of $G_L \times G_R$, i.e. the left- and right-handed currents, the invariance of (4.20) under this subgroup implies for the corresponding boundary state the condition

$$J^a = \left(Ad_{f^{-1}} \overline{J}\right)^a \tag{4.22}$$

which modifies (4.5) by conjugating the right-handed currents by f^{-1}.

5. D-branes in the near-horizon geometry of NS5-branes

After assembling the necessary tools, we are now ready to study the physics of the configurations of figure 4.

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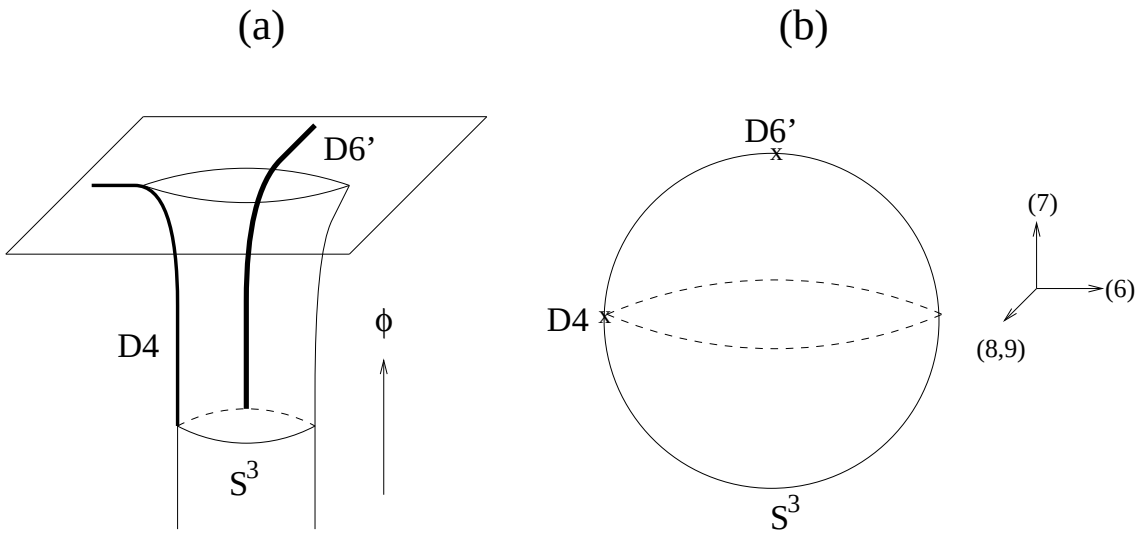

Figure 9: (a) D4- and D6'-branes on background of NS5-branes, (b) intersection with three-sphere.

5.1 D4- and D6′-branes ending on NS5-branes

We start with the configuration of figure 4*a*. A stack of N_c D4-branes ends from the left, i.e. from negative x^6, on k coincident NS5-branes. N_F D6′-branes end on the NS5-branes from above (positive x^7). From the point of view of the geometry (3.1) (figure 5), the D-branes extend into the CHS throat, as indicated in figure 9*a*.

The D4-branes intersect the three-sphere at the point $x^7 = x^8 = x^9 = 0$; the D6′-branes at $x^6 = x^8 = x^9 = 0$ (see figure 9*b*). Thus, (3.6), they correspond to the boundary states $g|_{\text{boundary}} = 1$ and $i\sigma_3$, respectively. The D4-branes are described by the boundary state with $\theta = 0$ and $f = 1$ (see (4.4) and (4.19)), while the D6′-branes correspond to a transformed state (4.20), with $\theta = 0$ and $f = \exp(i\pi\sigma_3/2) = i\sigma_3$. The SU(2) currents J^a satisfy the boundary conditions (4.5) and (4.22) for strings ending on the D4- and D6′-branes, respectively. In order to preserve worldsheet supersymmetry one has to impose analogous boundary conditions on the fermions. For example, for a string ending on the D4-branes one has

$$\chi^a = \overline{\chi}^a \tag{5.1}$$

while for a boundary on a D6′-brane the χ^a satisfy an analog of (4.22). Since, as is clear from figure 9, both the fourbranes and the sixbranes extend into the throat, the boundary conditions on ϕ are Neumann.

We would like to construct the vertex operator for emitting the lowest lying $4-6'$ string in the geometry of figure 9, i.e. generalize (4.1) to the fivebrane near-horizon geometry. Some parts of the discussion leading to (4.1) are unchanged. In particular, the geometry is the same as there in $(x^0, x^1, x^2, x^3, x^4, x^5)$. The presence of ϕ allows

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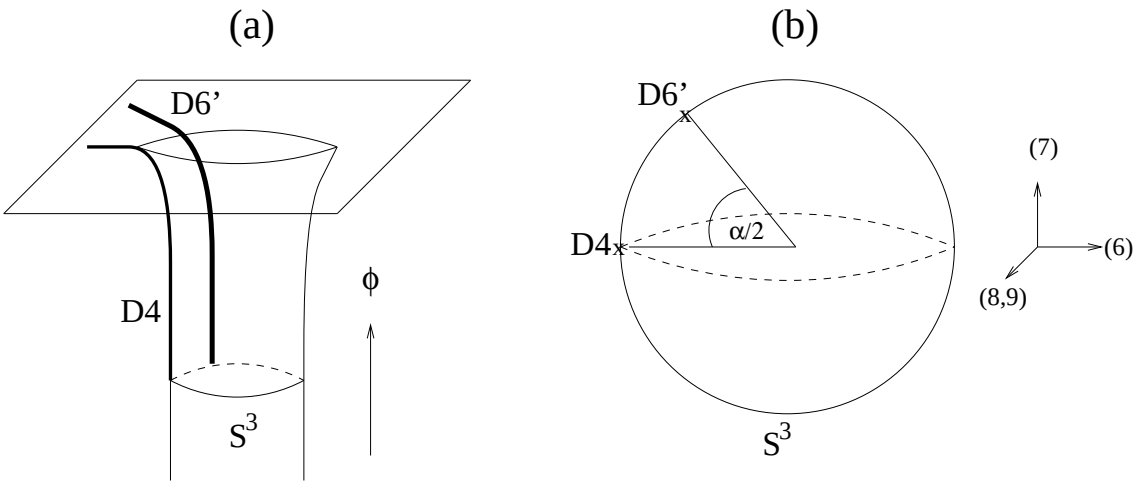

Figure 10: (a) D4- and D6'- branes on background of NS5-branes with a generic angle between them, (b) intersection with three-sphere.

also a contribution $\exp(\beta\phi)$ to the vertex operator. Thus, the analog of (4.1) for this case has the form

$$V = e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}e^{\beta\phi}V_2\,, \tag{5.2}$$

where V_2 is the contribution of the SU(2) group manifold to the vertex operator, to which we turn next.

The $4-6'$ vertex operator V_2 changes the worldsheet boundary conditions from $g = 1$ to $g = f = \exp(i\alpha\sigma_3/2)$. The D6′-brane corresponds to $\alpha = \pi$, but it is instructive to discuss the general case, in which the angle between the D4- and D6′-branes is $\alpha/2$ (see figure 10).

As discussed above, $g = 1$ corresponds to the boundary condition (4.5), (5.1) for the SU(2) currents and fermions, while $g = f$ gives rise to (4.22)

$$J^3 = \overline{J}^3\,, \qquad \chi^3 = \overline{\chi}^3\,, \qquad J^\pm = \exp(\mp i\alpha)\overline{J}^\pm\,, \qquad \chi^\pm = \exp(\mp i\alpha)\overline{\chi}^\pm\,. \tag{5.3}$$

Note that the symmetry generated by $J^3 + \bar{J}^3$ is preserved by both (4.5) and (5.3). As discussed after eq. (3.6), it corresponds to the rotation symmetry $\mathrm{SO}(2)_{89}$ which is unbroken by the brane configuration. As in section 4.1, we conclude that V_2 must include a twist field σ_α with the following locality properties w.r.t. the left-moving currents J^a and fermions χ^a:

$$\begin{aligned}
\sigma_\alpha(z)J^3(z') &\sim (z-z')^{m_1}O_1(z')\,,\\
\sigma_\alpha(z)\chi^3(z') &\sim (z-z')^{m_2}O_2(z')\,,\\
\sigma_\alpha(z)J^\pm(z') &\sim (z-z')^{\mp\frac{\alpha}{2\pi}+n_1}O_3(z')\,,\\
\sigma_\alpha(z)\chi^\pm(z') &\sim (z-z')^{\mp\frac{\alpha}{2\pi}+n_2}O_4(z')
\end{aligned} \tag{5.4}$$

with $m_i, n_i \in \mathbb{Z}$; O_I on the r.h.s. are operators whose precise form will not be specified here. A similar twist holds for the right-movers. Equation (5.4) implies

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that V_2 belongs to a representation of a twisted affine SU(2), in which the currents $J^\pm$ have fractional modes. This twisted algebra reads

$$\begin{aligned}
\left[J^3_n, J^3_m\right] &= n\frac{k_B}{2}\delta_{n,-m}\,, \\
\left[J^3_n, J^\pm_{m\pm\frac{\alpha}{2\pi}}\right] &= \pm J^\pm_{n+m\pm\frac{\alpha}{2\pi}}\,, \\
\left[J^+_{n+\frac{\alpha}{2\pi}}, J^-_{m-\frac{\alpha}{2\pi}}\right] &= \left(n+\frac{\alpha}{2\pi}\right)k_B\delta_{n,-m} + 2J^3_{n+m}\,,
\end{aligned} \tag{5.5}$$

where $k_B = k-2$ is the level of the bosonic SU(2) algebra (as in section 3). The algebra (5.5) can be mapped into the standard (untwisted) affine Lie algebra by using spectral flow. If J^a satisfy the twisted algebra (5.5), the generators $\tilde{J}$ defined by

$$\tilde{J}^3_n = J^3_n + \frac{k_B}{2}\frac{\alpha}{2\pi}\delta_{n,o}\,, \qquad \tilde{J}^\pm_n = J^\pm_{n\pm\frac{\alpha}{2\pi}} \tag{5.6}$$

satisfy the ordinary untwisted algebra. Thus one can use standard facts about the representations of untwisted affine Lie algebra to study the twisted one.

The modes of the energy-momentum tensor ($\tilde{L}_n$) constructed from $\tilde{J}$ are related to those of the original energy-momentum tensor (L_n) via

$$\tilde{L}_n = L_n + \frac{\alpha}{2\pi}J^3_n + \frac{k_B}{4}\frac{\alpha^2}{4\pi^2}\delta_{n,0}\,. \tag{5.7}$$

To understand the properties of the operator σ_α, consider first the case $\alpha = 0$. σ_0 describes an open string connecting two boundary states, both corresponding to $j = 0$ in eq. (4.19). As discussed in section 4.2, this open string should transform in a representation of the diagonal SU(2) contained in the fusion of two $j = 0$ representations, which consists of only the spin 0 representation. If h is the energy (L_0) and m the J^3 charge, then the lowest energy state in this sector has $h = m = 0$ and all the excited states satisfy

$$h \geq |m|\,. \tag{5.8}$$

Turning on α continuously, the open string created by σ_α remains in the spin 0 representation, now of the affine algebra generated by the currents $\tilde{J}$ (5.6). Hence the corresponding states satisfy

$$\tilde{h} \geq |\tilde{m}| \tag{5.9}$$

or, using (5.7),

$$h \geq \left(\pm 1 - \frac{\alpha}{2\pi}\right)\tilde{m} + \frac{k_B}{4}\frac{\alpha^2}{4\pi^2}\,, \tag{5.10}$$

where $\tilde{m} \in Z$. For $0 \leq \alpha < 2\pi$, the lowest energy state corresponds to $\tilde{m} = 0$. The dimension and charge of the corresponding operator are

$$h = \frac{k_B}{4}\frac{\alpha^2}{4\pi^2}\,, \qquad m = -\frac{k_B}{2}\frac{\alpha}{2\pi}\,. \tag{5.11}$$

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To construct this operator it is convenient to decompose the $\mathrm{SU}(2)_{k_B}$ bosonic CFT under $\mathrm{U}(1) \times \mathrm{SU}(2)/\mathrm{U}(1)$, where $\mathrm{U}(1)$ represents the subgroup generated by J^3. One can bosonize[8] J^3 as in (3.9), $J^3 = 2i\sqrt{k_B/2}\,\partial u$; $J^\pm$ are represented by $\exp(\pm 2i\sqrt{2/k_B}u)$ multiplied by operators from the $\mathrm{SU}(2)/\mathrm{U}(1)$ coset. The operators

$$\sigma_\alpha^B = \exp\left[-i\sqrt{\frac{k_B}{2}}\left(\frac{\alpha}{2\pi}+n\right)u\right] \tag{5.12}$$

with $n \in \mathbb{Z}$, have the right locality properties (5.4) with respect to the currents. The dimension and charge (5.11) correspond to those of the operator (5.12) with $n = 0$. Note that for $\pi \leq \alpha < 2\pi$, setting $n = -1$ in (5.12) would give a lower dimension than that of $n = 0$, however, the charge and dimension do not satisfy the inequality (5.10) in this case. Hence, these operators are not in the spectrum.

The operator σ_α must also flip the boundary conditions of the fermions χ^a from (5.1) to (5.3). This means that it must include a spin field for $\chi^\pm$. Bosonizing the two fermions,

$$i\partial H = \chi^+\chi^- \tag{5.13}$$

one finds that the operators with the right locality properties w.r.t. $\chi^\pm$ are

$$\Sigma_n = e^{i(n-\frac{\alpha}{2\pi})H}\,, \tag{5.14}$$

where $n \in \mathbb{Z}$. For $0 \leq \alpha < \pi$ the operator of lowest dimension out of these is Σ_0 whose scaling dimension is $h = \alpha^2/8\pi^2$ and $\mathrm{U}(1)$ charge $-\alpha/2\pi$. Thus, in this range of α, the lowest lying state in the sector twisted by α is

$$\sigma_\alpha = e^{-i\frac{\alpha}{2\pi}H}e^{-i\frac{\alpha}{2\pi}\sqrt{\frac{k_B}{2}}u}\,. \tag{5.15}$$

Notice that σ_α is a twist field for J_{total}, i.e. one can rewrite it as

$$\sigma_\alpha = e^{-i\sqrt{\frac{k}{2}}\frac{\alpha}{2\pi}Y}\,, \tag{5.16}$$

where Y is defined in (3.9) and is related to u and H via

$$\sqrt{\frac{k}{2}}Y = \sqrt{\frac{k_B}{2}}u + H\,. \tag{5.17}$$

The operators (5.14) with $n \neq 0$ can be thought of as "excited" twist fields. For $\pi < \alpha \leq 2\pi$ the operator Σ_1 in (5.14) has lower dimension then that of Σ_0. Then the excited twist operator

$$\sigma'_\alpha = e^{-i(\frac{\alpha}{2\pi}-1)H}e^{-i\sqrt{\frac{k_B}{2}}\frac{\alpha}{2\pi}u} \tag{5.18}$$

[8] As in section 4.1, here and below scalar fields are canonically normalized on the boundary; e.g. $\langle u(z)u(w)\rangle = -\log|z-w|$ for $z, w \in \mathbb{R}$.

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has the smallest dimension in the α twisted sector.[9] For $\alpha = \pi$, the supersymmetric case, the two operators in (5.15) and in (5.18) are degenerate on the worldsheet. Nevertheless, we show in appendix B that the operator (5.18) creates from the vacuum excited $4-6'$ strings, even for $\alpha = \pi$.

To summarize, the lowest lying open string connecting the D4- and D6′-branes in the configuration of figure 10 (which we will refer to as a $4-6'_+$ string, since it connects the fourbrane to a half-infinite D6′-brane at $x^7 > 0$) is described by the vertex operator

$$V^+_{4-6'} = e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}e^{\beta\phi}e^{-i\frac{\alpha}{2\pi}\sqrt{\frac{k}{2}}Y}\,. \tag{5.19}$$

The mass shell condition requires that

$$\frac{1}{2}k_\mu^2 - \frac{1}{2}\beta(\beta+Q) + \frac{k}{4}\left(\frac{\alpha}{2\pi}\right)^2 = \frac{1}{4}\,. \tag{5.20}$$

We see that the $4-6'_+$ string vertex (5.19) describes an on-shell excitation in *five* non-compact dimensions, $(x^0, x^1, x^2, x^3, \phi)$. This is clearly the same phenomenon as that discussed for closed strings in section 3. The extra non-compact dimension is the infinite throat of [4] associated with the radial direction. Non-normalizable open string vertex operators such as (5.19) with $\beta > -Q/2$ correspond to *off-shell observables* in the theory on the D-branes.

To find the spectrum of low lying on-shell states, which is relevant for analyzing the physics of the brane configurations described in section 2, one has to find the normalizable states corresponding to $4-6'_+$ strings. One way of doing that is to compute the correlation functions of the non-normalizable operators (5.19) and extract the spectrum by analyzing their analytic structure. As explained in section 3, to do this within a weakly coupled string theory one has to regularize the strong coupling region $\phi \to -\infty$. Following [13], we will do this by considering the geometry (3.10) corresponding to fivebranes separated along a circle in the (x^6, x^7) plane.[10]

The effect of this regularization on the vertex operator (5.19) is to change the wavefunction on the infinite cylinder labeled by (ϕ, Y) to a wavefunction on the semi-infinite cigar,

$$e^{\beta\phi}e^{-i\frac{\alpha}{2\pi}\sqrt{\frac{k}{2}}Y} \longrightarrow V_{jm}\,, \tag{5.21}$$

where

$$j = \beta\sqrt{\frac{k}{2}}\,, \qquad m = -\frac{k}{2}\frac{\alpha}{2\pi} \tag{5.22}$$

[9] Applying the GSO projection to (5.2) will pick up different spin fields S_{45} for the different twist fields (5.15) and (5.18).

[10] One might wonder whether the separation of the fivebranes introduces a finite mass shift for $4-6'$ strings, when the fourbrane and sixbrane end on different fivebranes. To see that this does not happen note that, as discussed in [13], the cigar geometry corresponds to the limit $g_s, r_0/l_s \to 0$ with $r_0/(l_s g_s)$ fixed and large (r_0 is the typical separation between fivebranes). The mass shift for $4-6'$ strings associated with this separation is $\sim r_0/l_s^2$; it goes to zero in string units in the above limit.

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j can be thought of as momentum along the cigar, while m is the momentum around the cigar (both the D4-branes and D6′-branes are wrapping the cylinder, and in particular the circle labeled by Y).

The vertex operator (5.19) becomes in the geometry (3.10)

$$V^+_{4-6'} = e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}V_{jm} \tag{5.23}$$

with (5.20)

$$\frac{k_\mu^2}{2} + \frac{m^2 - j(j+1)}{k} = \frac{1}{4}\,. \tag{5.24}$$

To compute the spectrum of low-lying normalizable excitations of the $4-6'_+$ string, one computes the two point function of the operators (5.23) on the disk. This amplitude exhibits first order poles in j. Using the mass-shell condition (5.24), these can be interpreted as poles in k_μ^2; they correspond to on-shell particles in four dimensions, created from the vacuum by the operator (5.23).

The calculation of this two point function is very similar to its closed string analog [17, 13], and is described in appendix A. The result is, as in the closed string case, a series of poles corresponding to the discrete representations of SL(2), (3.15). The lowest lying state corresponds to $n = 1$, i.e. $j = |m| - 1$. Plugging this together with (5.22) into (5.24) we find that the mass of the lowest lying normalizable state of the $4-6'_+$ string is

$$M^2(\alpha) = \frac{1}{2}\left(\frac{\alpha}{\pi} - 1\right). \tag{5.25}$$

In particular, we find that as expected, for $\alpha = \pi$ the lowest lying state is massless. The vertex operator which creates this massless particle from the vacuum is

$$V^+_{4-6'}\left(k_\mu^2 = 0\right) = e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}V_{\frac{k}{4}-1,-\frac{k}{4}}\,. \tag{5.26}$$

Note that the degeneracy is correct too. The spin field S_{45} in (5.23) takes apriori two possible values, but the GSO projection picks one of them. Thus, (5.23) describes a single complex scalar field Q which transforms in the (N_c, N_F) of the $\mathrm{U}(N_c)\times\mathrm{U}(N_F)$ symmetry on the D-branes, as explained in section 2. Applying the spacetime supercharges gives rise to a chiral superfield[11] Q. The complex conjugate field $Q^\dagger$ arises from $4-6'_+$ strings with the opposite orientation (a $6'_+ - 4$ string). The field $\tilde{Q}$ ($\tilde{Q}^\dagger$) transforming in the $(\overline{N}_c, \overline{N}_F)$ representation that is needed for anomaly cancellation of $N = 1$ SQCD comes from $6'_- - 4$ ($4-6'_-$) strings connecting the fourbranes to sixbranes attached to the NS5-branes from below, as anticipated in [7]. These are de-

[11] In particular, one can check that (5.23), (5.26) is annihilated by half of the spacetime supercharges.

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scribed by vertex operators of the form (5.19), (5.23) and (5.26) with $\alpha < 0$ and with the $S^\dagger_{45}$ spin field for the $4-6'_-$ operator $\tilde{Q}^\dagger$ (replacing S_{45} for the $4-6'_+$ operator Q).[12] Some comments are in order at this point:

(1) The above analysis is valid as long as long as $j = |m| - 1 > -1/2$ satisfies the unitarity bound (3.14), i.e. for $\alpha > \alpha_c = 2\pi/k$. For $\alpha < \alpha_c$, the lowest lying normalizable states belong to the continuum ($j = -1/2 + is$) and the physics is different.

(2) An interesting question is whether m, (5.22), is quantized on the cigar. Before separating the NS5-branes on a circle, it is clear that α in (5.19) (and hence m) is arbitrary — the angle between the D4- and D6′-branes is not constrained. After the separation the situation is different. The D-branes are wrapped around the cigar, and the momentum around the cigar m appears to be quantized, $m \in \mathbb{Z}$. For D-branes wrapped around a circle one can shift the momentum by an arbitrary fractional amount, by turning on a Wilson line of the gauge field on the branes (around the circle). However, on the cigar there are no non-contractible cycles, and hence one expects the quantization of m to persist.

(3) The quantization of m can be seen in other ways as well. One is to use a T-dual description, in which the cigar is replaced by $N = 2$ Liouville [13, 41, 42], and the D-branes are at points on the dual S^1. The $N = 2$ Liouville superpotential effectively pins down the (T-dual) field Y (3.9) to points on the circle when one goes far down the $N = 2$ Liouville throat to the strong coupling region. Since the D-branes extend into the throat and lie at points (independent of ϕ) on the circle, their positions on the circle must coincide with those determined by the superpotential. Another way to see the quantization is to note that quantization of α has a natural geometric interpretation when the branes are ending on fivebranes which lie on a circle. Since the D4- and D6′-branes point towards the center of the circle on which the fivebranes lie, the angle between the D-branes, $\alpha/2$, must be an integer multiple of $2\pi/k$ — the angular separation between the k fivebranes. This leads to the same conclusion, (5.22), $m \in \mathbb{Z}$. Thus, the D-branes seem to know that the smooth cigar is in fact associated with k fivebranes on a circle.

(4) It is interesting to compare the symmetry properties of (5.26) to those expected of Q, $\tilde{Q}$. The vertex operator $V^+_{4-6'}$ is charged under the unbroken rotation symmetry $\mathrm{SO}(2)_{45} \times \mathrm{SO}(2)_{89}$ of the configuration of figure 10. The $\mathrm{SO}(2)_{45}$ charge

[12] The reason for the $S^\dagger_{45}$ is the following. The D6′$_-$ and D6′$_+$-branes must have the same orientation. They can be obtained by starting with parallel D6′- and $\bar{\mathrm{D}}$6′-branes (at $\alpha = 0$). The D6′$_+$-brane is obtained by rotating the D6′-brane by an angle $\alpha/2 = \pi/2$, while the D6′$_-$-brane is obtained by rotating the $\bar{\mathrm{D}}$6′-brane by an angle $\alpha/2 = -\pi/2$. The conjugation of S_{45} in passing from the D6′ to the $\bar{\mathrm{D}}$6′-brane is due to the different GSO projection on branes and anti branes.

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is carried by the spin field S_{45} in (5.26). In units in which the supercharges have $SO(2)_{45}$ charge $\pm 1/2$, $V^+_{4-6'}$ has charge $1/2$. The $SO(2)_{89}$ charge of (5.19) is carried by the last factor, $\exp(-i\frac{\alpha}{2\pi}\sqrt{\frac{k}{2}}Y)$. Normalizing it such that the supercharges again have charge $\pm 1/2$, one finds that the charge of $V^+_{4-6'}$ is $-\frac{k\alpha}{4\pi}$. Therefore, the $SO(2)_{89}$ charge of (5.26) is $-k/4$. From the discussion above one learns that $\tilde{Q}$ carries the same charge as Q. These assignments are similar but not identical to those postulated in brane theory in the past. In our notation, the $SO(2)_{45} \times SO(2)_{89}$ charge of $Q, \tilde{Q}$ was postulated to be $(1/2, 0)$ (see e.g. [1], discussion between eqs. (182) and (183)). In the present construction, we find charge $(\frac{1}{2}, -\frac{k\alpha}{4\pi})$ for Q and $\tilde{Q}$. The discrepancy in the $SO(2)_{89}$ charges might be related to that found in [13]; a better understanding of its origin is left for future work.

For $\alpha < \pi$ the lowest lying state (5.25) is tachyonic; the stable vacuum is obtained by its condensation. This process has a very natural interpretation from the point of view of brane theory, which also makes it clear what is the endpoint of the condensation. For $\alpha \neq \pm\pi$, the configuration of figure 10 is not supersymmetric, hence stability needs to be checked. For $\alpha < \pi$ the configuration of figure 10 can reduce its energy by having the D4-brane slide away from the NS5-brane so that it ends on the D6′-brane instead (figure 11). The resulting vacuum is stable. For $\alpha > \pi$, the configuration of figures 10 and 11 is stable under small deformations. Indeed, the lowest lying open string state is massive in this case (5.25).

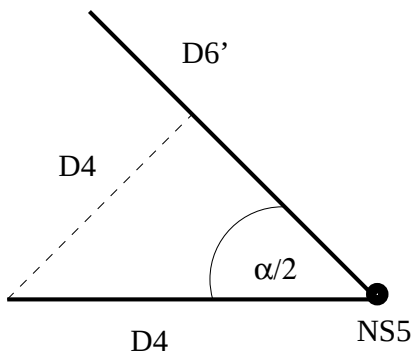

Figure 11: Stable configuration for D4-D6' system after tachyon condensation.

From the point of view of the brane configurations describing four-dimensional gauge theories like that of figure 3, one can change α from π in a number of ways. One is to change the relative position of the NS5- and NS5′-branes in x^7 by the amount Δx^7. In the gauge theory on the D4-branes this corresponds to turning on a Fayet-Iliopoulos D-term [6, 1]. Depending on the sign of the FI term, either Q or $\tilde{Q}$ should condense to minimize the D-term potential

$$V_D \sim \left(Q^\dagger Q - \tilde{Q}^\dagger \tilde{Q} - r\right)^2, \tag{5.27}$$

where r is proportional to Δx^7. The gauge theory analysis is nicely reproduced by our string theory considerations.[13] $\Delta x^7 > 0$ corresponds to $0 < \alpha < \pi$. In this case the ground state of the $4 - 6_+$ string is tachyonic, Q condenses and the D4-branes

[13]Both in gauge theory and in string theory, in the context of the full configuration of figure 3 the foregoing discussion is valid for $N_f \geq N_c$ (see [1] for details).

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detach from the NS5′-branes and attach to the D6-branes. $\Delta x^7 < 0$ corresponds to $\alpha > \pi$ for the $4-6_+$ string which is hence massive, but since $0 < -\alpha < \pi$ for the $4-6_-$ strings ($\tilde{Q}$), a similar process of condensation to the above occurs for them.

Another way of changing α is to tilt the D6-branes by some angle in the (x^6, x^7) plane, which must lead to a potential similar to (5.27).
More comments on the supersymmetric case $\alpha = \pi$:

(1) Like in [13], for values of j that satisfy the unitarity constraint (3.14), the full spectrum obtained from (3.15) and (5.24) and its generalization to other observables is non tachyonic.

(2) The excited twist field (5.18) does not create from the vacuum massless states; this is shown in appendix B.

(3) Massive states form hypermultiplets, as they should; this is also shown in appendix B.

(4) Like in the closed string case [3, 13], there is a continuum of δ-function normalizable $4-6'$ string states corresponding to $j \in -1/2 + i\mathbb{R}$. These states are separated by an energy gap of order $1/l_s$ from the massless discrete states discussed above.

5.2 D4-branes ending on NS5-branes from opposite sides

We next turn to the configuration of figure 4*b*. A stack of N_L D4-branes ends from the left on k coincident NS5-branes, and a stack of N_R D4-branes ends on the NS5-branes from the right. In this case, in the parametrization (3.6), the D4-branes on the left intersect the group manifold at $g = 1$ while the ones on the right intersect it at $g = -1$ (see figure 12).

We first consider the case where the two stacks of D4-branes are at the same point in the (x^4, x^5) plane. To construct the vertex operator for emitting the lowest lying $4_L - 4_R$ string in the geometry of figure 12, we can follow the discussion of the previous subsection. The dependence on $(x^0, x^1, x^2, x^3, \phi)$ is the same as in the $4-6'$ case. In the (x^4, x^5) directions we now have Dirichlet-Dirichlet boundary conditions. Therefore, the $4-4$ vertex operator is:

$$V_{4-4} = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} V_2 \,. \tag{5.28}$$

The contribution V_2 of the SU(2) SCFT to (5.28) is the same as in section 5.1, with $\alpha = \pm 2\pi$:

$$V_2^{\pm} = \sigma_{\mp 2\pi} = e^{\pm i\sqrt{k/2}Y} \,. \tag{5.29}$$

Hence

$$V_{4-4}^{\pm} = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} e^{\pm i\sqrt{k/2}Y} \,. \tag{5.30}$$

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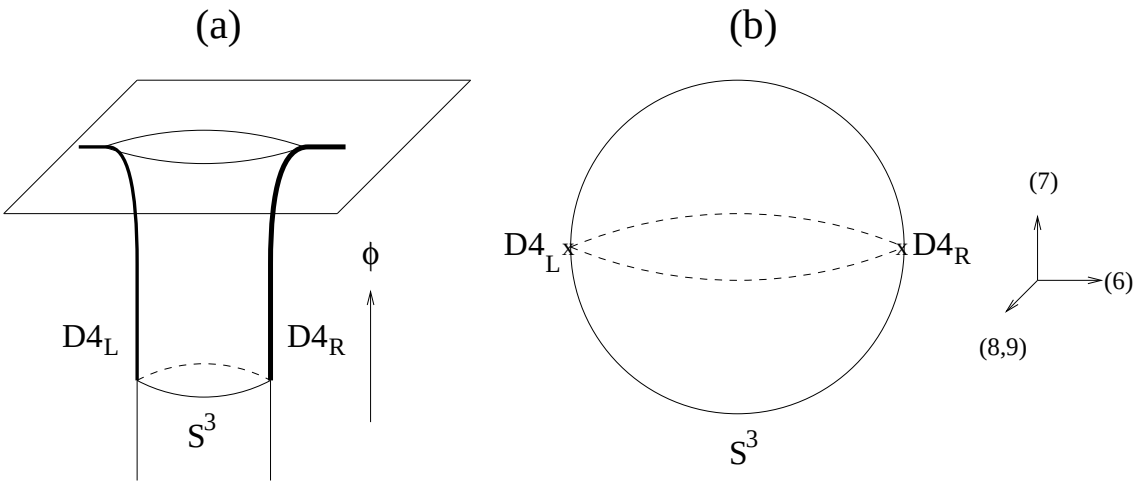

Figure 12: (a) $D4_L$ and $D4_R$ on background of NS5-branes, (b) intersection with three-sphere.

To describe $V^{\pm}_{4-4}$ in the cigar background (3.10), we use eqs. (5.21) and (5.22), with $\alpha = \mp 2\pi$. The vertex operators (5.30) become in this geometry:

$$V^{\pm}_{4-4} = e^{-\varphi} e^{ik_\mu x^\mu} V_{j,\pm k/2}\,. \tag{5.31}$$

The mass shell condition reads:

$$\frac{k_\mu^2}{2} + \frac{k}{4} - \frac{j(j+1)}{k} = \frac{1}{2} \Longrightarrow -k_\mu^2 = \frac{2}{k}\left[\left(\frac{k}{2}-1\right)\frac{k}{2} - j(j+1)\right]. \tag{5.32}$$

As in [13] and section 5.1, the two point function of (5.31) has a series of poles corresponding to discrete representations of SL(2), and the lowest lying state corresponds to

$$j = |m| - 1 = \frac{k}{2} - 1\,. \tag{5.33}$$

Plugging (5.33) into (5.32) we find that in this case $V^{\pm}_{4-4}$ create massless states from the vacuum.

The vertex operators which couple to these massless particles take the form (5.31) and (5.33):

$$V^{\pm}_{4-4}\left(k_\mu^2 = 0\right) = e^{-\varphi} e^{ik_\mu x^\mu} V_{k/2-1,\pm k/2}\,. \tag{5.34}$$

As expected from gauge theory, $V^{\pm}_{4-4}$ describe two complex scalar fields Q, $\tilde{Q}^\dagger$, in the (N_L, N_R) of the $\mathrm{U}(N_L) \times \mathrm{U}(N_R)$ symmetry on the D-branes, respectively. The complex conjugates $Q^\dagger$, $\tilde{Q}$ arise from $4_L - 4_R$ strings with the opposite orientation.

In this case, the charges of $Q, \tilde{Q}$ under the $\mathrm{SO}(2)_{45} \times \mathrm{SO}(2)_{89}$ R-symmetry are $(0, k/2)$. The $\mathrm{SO}(2)_{45}$ agrees with expectations. The $\mathrm{SO}(2)_{89}$ charge predicted by brane theory is $1/2$, which is the value obtained by extrapolating our results to $k = 1$

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(which, as discussed above, is in fact outside the range of validity of our approach). For $k > 1$, the situation is less clear, but it is worth pointing out that in addition to the fact that the same issue already arose in the previous subsection and in [13], it also appeared in brane theory before. As reviewed in [1], the theory on D4-branes ending on k NS5-branes typically contains chiral superfields Φ with a polynomial superpotential $s_0\Phi^{k+1}$. The coupling s_0 appears to be charged under the analog of $SO(2)_{89}$. The resolution of all these problems is again left for future work.
Comments:

(1) If the N_L D4-branes are located at $x^4 = x^5 = 0$, and the N_R D4-branes are at $(x^4, x^5) = (a, b)$, one can apply the discussion of section 4.1. As in (4.1), the vertex operators $V^{\pm}_{4-4}$ have to be multiplied by the "winding" generating factor $\exp\{\frac{i}{\pi}[a(x^4_L - x^4_R) + b(x^5_L - x^5_R)]\}$. The lowest lying $4_L - 4_R$ states have mass squared $\frac{1}{\pi^2}(a^2 + b^2)$, in agreement with expectations.

(2) For the case $\alpha = \pm 2\pi$, the SU(2) current algebra is untwisted (see (5.3)). The discussion of section 4.2 shows that the $D4_L$- and $D4_R$-branes correspond in this case to the boundary states with $j = 0$ and $j = k_B/2$, respectively. Hence, the strings that connect the two transform in the $j = k_B/2$ representation of the bosonic $\widehat{SU(2)}$. Denoting the corresponding vertex operators by $\sigma_{k_B/2,m}$ ($m = -k_B/2, -k_B/2+1, \ldots, k_B/2-1, k_B/2$), we find that V_2 in (5.28) and (5.29) is given by

$$V_2^{\pm} = e^{\pm i\sqrt{k/2}Y} = \chi^{\pm}\sigma^B_{\mp 2\pi} = \chi^{\pm}\sigma_{k_B/2,\pm k_B/2}\,. \tag{5.35}$$

The last expression can be thought of as the highest and lowest weight states in a spin $k/2 = k_B/2 + 1$ representation of the total SU(2). The full set of primaries at the lowest level of $4_L - 4_R$ strings is the following:

$$\begin{aligned}
&\text{(i)} \quad e^{-\varphi}e^{ik_\mu x^\mu}e^{\beta\phi}\left(\chi\cdot\sigma_{k_B/2}\right)_{j=k/2}\,,\\
&\text{(ii)} \quad e^{-\varphi}e^{ik_\mu x^\mu}e^{\beta\phi}\left(\chi\cdot\sigma_{k_B/2}\right)_{j=k/2-2}\,,\\
&\text{(iii)} \quad e^{-\varphi}e^{ik_\mu x^\mu}e^{\beta\phi}\left[\sum_M a_M\psi^M\sigma_{k_B/2} + a_8\left(\chi\cdot\sigma_{k_B/2}\right)_{j=k/2-1}\right].
\end{aligned} \tag{5.36}$$

In (i) and (ii), χ and $\sigma_{k_B/2}$ are coupled to a representation of total spin $k/2$ and $k/2-2$, respectively. (iii) contains six physical combinations (after imposing the physical state conditions on the polarization coefficients a_M, and eliminating null states); $\{\psi^M\} = \{\psi^\mu, \psi^4, \psi^5, \chi^r\}$. It can be shown that out of the operators (5.36) only those whose SU(2) part is (5.35) create massless particles (see appendix C).

(3) One can study $4_L - 4_R$ strings when the angle between the two kinds of branes is generic, $\alpha/2$. For $\alpha/2 \neq \pm\pi$, an analysis similar to the one in section 5.1 shows that the lowest lying $4_L - 4_R$ states are tachyonic. For $\alpha = 0$, one finds a system of parallel $D4 - \bar{D}4$. This is discussed in the next subsection.

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5.3 Rotating $D4-D4$ systems into $D4-\bar{D}4$

The vertex operators creating the low lying states of $4_L - 4_R$ strings when the angle between the $D4_L$- and $D4_R$-branes is $-\pi < \alpha/2 < \pi$ are (see subsection 5.1):

$$V_{4-4}(\alpha/2) = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} e^{-i\frac{\alpha}{2\pi}\sqrt{k/2}Y} \,. \tag{5.37}$$

Repeating the analysis of sections 5.1 and 5.2, one finds that the lowest lying states that (5.37) creates from the vacuum are tachyonic. At $\alpha = 0$, when the two D4-branes are anti parallel,

$$V_{4-4}(0) = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} \tag{5.38}$$

is the tachyon vertex operator creating the low lying state of a string connecting a D4-brane parallel to an anti-D4-brane ($\bar{D}4$-brane), both ending on the NS5-branes from the same side; it is a $4_L - \bar{4}_L$ string.

As before, the tachyon signals that the D4-branes could reduce their energy by reconnecting. Imagine for simplicity that the ends of the D4-branes are pinned down far from the NS5-branes (see figure 13). Tachyon condensation corresponds to a process where the D4-branes (solid lines) connect to each other and detach from the fivebrane. They can then reduce their energy by stretching straight between the two pinned endpoints (dashed line). As is clear from figure 13, their energy decreases in the process. As $\alpha \to 0$, the fourbranes annihilate (or more generally become lower-dimensional branes). This generalizes the results of [43] on brane annihilation to D-branes ending on fivebranes.

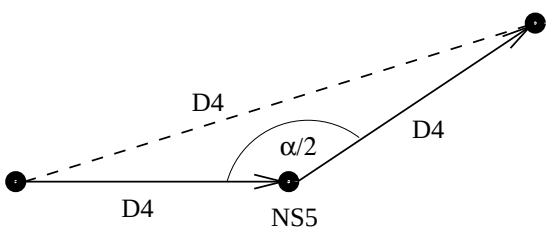

Figure 13: Two D4-branes at a generic angle between them and their corresponding stable configuration.

Similarly, the vertex operator creating the low lying states of $4_L - 4_L$ strings when the angle between the $D4_L$-branes is $\alpha/2$ is

$$V_{4-4}\left(\frac{\alpha}{2}\right) = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} e^{i[(\frac{\alpha}{|\alpha|}-\frac{\alpha}{2\pi})H - \sqrt{\frac{k_B}{2}}\frac{\alpha}{2\pi}u]} \tag{5.39}$$

(the last exponent is the excited twist field (5.18)). The extra factor of $\exp(\pm iH)$ in (5.39) compared to (5.37) is due to the different signs of the GSO projection in the two cases resulting from the reversed orientation of the $D4_R$-brane. At $\alpha = 0$, we obtain the $4_L - 4_L$ vertex operator which in the cigar geometry is (recall (5.13) and see appendix B for notation)

$$V^{\pm}_{4-4}(0) = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} \chi^{\pm} \longrightarrow e^{-\varphi} e^{ik_\mu x^\mu} z_1^{k-2} V_{j,\pm 1} \tag{5.40}$$

(the $\pm$ in (5.40) is correlated with $\alpha \to 0_+$ or $\alpha \to 0_-$ in (5.39)). At first sight one might be puzzled why for two parallel D4-branes ending on a stack of NS5-branes one does not find additional massless particles. For example, in section 2 it was argued

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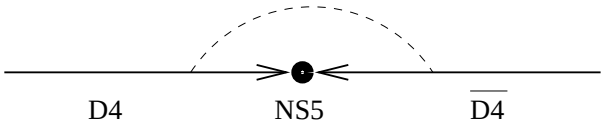

Figure 14: A configuration of D4 and $\bar{\text{D}}4$ at the opposite sides of NS5-brane.

that fundamental strings stretching between the fourbranes give rise to massless non-abelian gauge bosons on the fourbranes. The reason we are not supposed to see those here is that we are studying the physics of the D-branes in the near-horizon geometry of the fivebranes. Only states whose wavefunctions are bound to the fivebranes can give rise to normalizable modes in our analysis. The wavefunctions of the gauge bosons discussed in section 2 are in contrast spread out in x^6 and, in particular, are not localized at the fivebranes. From the point of view of the near-horizon geometry, they are non normalizable.

At $\alpha/2 = \pm\pi$ (5.39) turns into

$$V_{4-\bar{4}}(\pm\pi) = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} \sigma^B_{\pm 2\pi} \tag{5.41}$$

which is the tachyon vertex operator for a $4_L - \bar{4}_R$ string connecting a D4-brane and a $\bar{\text{D}}4$-brane located on opposite sides of the stack of NS5-branes (see figure 14).

Brane constructions involving D4- and $\bar{\text{D}}4$-branes ending on NS5-branes were recently studied in [44, 45]. The present work can be used to shed more light on such configurations.

5.4 A D6-brane intersecting a stack of NS5-branes

We now turn to the configuration of figure 4*c*. A stack of D4-branes ends from the left on k coincident NS5-branes, and a D6-brane intersects the NS5-branes at the same value of (x^4, x^5) as the fourbranes.

From the brane geometry point of view in type IIA, the notion of putting a D6-brane on top of k *coincident* NS5-branes is not well defined, due to the HW transition [6]. For instance, for a single NS5-brane one must specify if the D6-brane is located to the left of the NS5-brane or to its right. In the first case, the D6-brane can be moved freely away from the NS5-brane to the left in the x^6 direction, but if moved away to the right an extra D4-brane is created, stretched between the D6-brane and the NS5-brane. In the second case, the D4-brane will be created when moving the D6-brane to the left. The configuration space of this system is thus separated into two disconnected components.

Similarly, for the case of k NS5-branes one has to specify which of these fivebranes are to the left of the D6-brane and which of them are located to its right. If $n \leq k$ fivebranes are to the left of the D6-brane, n D4-branes will be created when the D6-brane moves to the left away from the stack of NS5-branes, and $k - n$ D4-branes

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will be formed if it is taken away to the right. Thus the configuration space consists of $k+1$ sectors — there are $k+1$ possibilities of what is meant by placing a D6-brane on top of k coincident fivebranes.

This ambiguity in type-IIA string theory can be understood in M-theory on $\mathbb{R}^{10} \times S^1$ when the radius of the S^1 is large in Planck units [46]. The D6-brane becomes a Kaluza-Klein monopole, i.e. a bundle whose fiber is the eleventh circular coordinate x^{10} of radius R_{10}. The bundle is non trivial and two patches are needed to trivialize it. These patches can be chosen as the two halves of the ten-dimensional space, one with $x^6 \geq 0$ to the right of the sixbrane and the other with $x^6 \leq 0$ to its left. Denote by $x^{10}_{\pm}$ the fiber coordinate over the two different patches. If ϑ is the azimuthal angle in the (x^4, x^5) plane, i.e. $\tan(\vartheta) = x^5/x^4$, then the transition between the fiber coordinates in the two patches on the transition (x^4, x^5) plane at $x^6 = 0$ is: $x^{10}_+/R = x^{10}_-/R_{10} + \vartheta$. From eleven-dimensional standpoint, an NS5-brane is an M5-brane which is a point in x^{10}, stretching along the (x^4, x^5) plane. Starting with k NS5-branes in presence of a D6-brane, one must specify which of these M5-branes have a definite x^{10}_+ coordinate, and which have a well defined x^{10}_- coordinate.

The division of configuration space into sectors is visible also in the worldsheet CFT description of section 4. Unlike the D6′-brane case studied in section 5.1, the D6-brane intersects the group manifold (3.6) not at a point but along a two-sphere of constant x^6. This corresponds to the conjugacy class $\mathbb{C}_\theta$ (4.4), where θ satisfies $\cos(\theta/2) = x^6/|\vec{x}|$. In section 4 we have seen that consistent boundary conditions of this type exist only for $k-1$ values of θ given by eq. (4.19). The D6-brane is thus allowed to intersect the stack of k NS5-branes only at fixed quantized values of x^6, namely, on two-spheres with quantized sizes S^2_n, $n = 1, 2, \ldots, k-1$ (see figure 15).

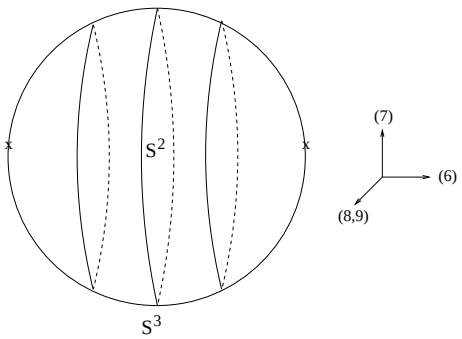

Figure 15: Conjugacy classes as possible positions of D6-branes.

It is natural to identify these $k-1$ different boundary states with the various possible positions of the D6-brane among the k NS5-branes, discussed above. There we had $k+1$ possible states, two of which are absent in the CFT approach. A possible interpretation of this fact is that when the D6-brane is either to the right or to the left of all NS5-branes, it is not bound to the fivebranes and hence does not give rise to a boundary state in the CHS geometry.

Anyhow, we can again study the massless $4-6$ open strings when the D6-brane is characterized by $\theta = 2\pi\frac{l}{k_B}$ in (4.19), with $2l = 0, 1, \ldots, k_B$. Such a string connects the $g = 1$ boundary state with $l = 0$, corresponding to the D4-brane, to a spin l boundary state corresponding to the D6-brane (see figure 16). The $4-6$ string belongs to representations contained in the fusion of spin 0 and spin l, i.e. the spin l representation. This consists of $2l+1$ operators σ_{lm}, $m = -l, -l+1, \ldots, l-1, l$,

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with scaling dimension

$$h(\sigma_{lm}) = \frac{l(l+1)}{k}\,. \tag{5.42}$$

As in subsection 5.2, we get for each l the vertex operators:

$$V_{lm} = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi}\,(\chi\cdot\sigma_l)_{l+1,m} \tag{5.43}$$

and some additional operators that do not create massless states. The two operators V_{lm} (5.43) with $m = \pm(l+1)$ do give rise to poles at $k_\mu^2 = 0$ in amplitudes and thus create massless particles of charge $\pm(l+1)$.

Recently, it was argued that n D0-branes in an SU(2) WZW background, which are n points on S^3, can turn into a single D2-brane which wraps a two-sphere with a quantized radius S_n^2 [38, 39]. This meshes nicely with the discussion above. In the near horizon of k NS5-branes, a D4-brane is a point on S^3 while a D6-brane wraps an $S_n^2 \in S^3$, $n = 1, 2, \ldots, k-1$. Hence, n D4-branes can condense into a single D6-brane in the $\theta = \pi\frac{n-1}{k-2}$ conjugacy class (4.19), namely, a D6-brane wrapping the sphere S_n^2. This is the HW transition. The D6-brane on S_n^2 is located, say, to the left of n out of the k NS5-branes. Moving it to the right, past these n fivebranes, creates n D4-branes stretched from the D6-brane to each of the n fivebranes. In the near horizon limit these look like n points on S^3.

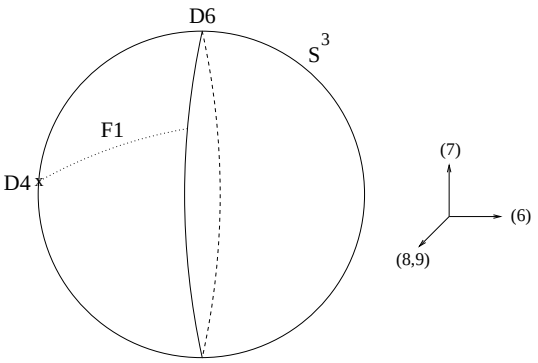

Figure 16: 4-6 open string on background of NS5-branes.

Acknowledgments

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A. $\langle V_{jm} V_{j,-m}\rangle$ on the disc

We first present some useful formulae:

$$C(a,b) = \int_{-\infty}^{\infty} |x|^{a-1}|1-x|^{b-1}dx = B(a,1-a-b) + B(a,b) + B(1-a-b,b)\,. \tag{A.1}$$

Here B is the Euler beta function:

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$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\,, \tag{A.2}$$

where

$$\Gamma(a+1) = a\Gamma(a)\,, \qquad \Gamma(-n+\epsilon) = \frac{(-)^n}{n!\epsilon} + \mathrm{O}(1)\,, \quad n = 0,1,2,\ldots. \tag{A.3}$$

To find the analytic structure of the two point functions of boundary operators discussed in this work, like (5.23), we need to compute the two point functions $\langle V_{jm} V_{j,-m}\rangle$ on the disc. This is the purpose of this appendix.

The calculation is similar to its closed string analog [17, 13], which we follow below. The CFT on the cigar is obtained by the coset construction from that on AdS_3. The natural observables in CFT on the euclidean version of AdS_3 on the disc are functions $\Phi_j(x;z)$ which transform as primaries under the diagonal (left + right) SL(2) current algebra (see section 4.2). x is an auxiliary real variable.

The two point function of Φ_j is[14]

$$\langle \Phi_j(x)\Phi_j(x')\rangle = A(j,k)|x-x'|^{-2(j+1)}\,, \tag{A.4}$$

where $A(j,k)$ is an analytic function of j in the domain (3.14); its precise form will not play a role below.

To study the coset it is convenient to "Fourier transform" the fields $\Phi_j(x)$ and define the mode operators

$$\Phi_{jm} = \int_{-\infty}^{\infty} \Phi_j(x)|x|^{j+m}dx\,. \tag{A.5}$$

The two point functions of the modes Φ_{jm} are equal to those of the $\mathrm{SL}(2)/\mathrm{U}(1)$ coset theory:

$$\langle V_{jm} V_{j,-m}\rangle = \langle \Phi_{jm}\Phi_{j,-m}\rangle\,. \tag{A.6}$$

Using (A.6), (A.5) and (A.4) one finds that

$$\langle V_{jm} V_{j,-m}\rangle = A'(j,k)\int_{-\infty}^{\infty} |y|^{j-m}|1-y|^{-2(j+1)}dy = C(j-m+1,-2j-1)\,. \tag{A.7}$$

Using eqs. (A.1), (A.2) and (A.3) one can finally express the two point functions in terms of gamma functions as:

$$\langle V_{jm} V_{j,-m}\rangle = A'(j,k)\left(\frac{\Gamma(j-m+1)\Gamma(j+m+1)}{\Gamma(2j+2)} + \frac{\Gamma(j-m+1)\Gamma(-2j-1)}{\Gamma(-j-m)} + \right.$$
$$\left. + \frac{\Gamma(j+m+1)\Gamma(-2j-1)}{\Gamma(-j+m)}\right)\,. \tag{A.8}$$

Using eq. (A.3) we can now find the analytic structure of $\langle V_{jm} V_{j,-m}\rangle$: it has single poles for j,m satisfying (3.15).[15]

[14] Here and below we suppress the dependence of correlation functions on the worldsheet location z on the real line, the boundary of the upper half plane.

[15] For $j = m$ one finds that the three terms in (A.8) conspire to give 0. This implies that the extra poles at $j = m$, found in some special cases in [19], appear on the sphere but not on the disc.

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B. Multiplicity of 4-6' open strings

In this appendix we show that the excited twist field (5.18) does not give rise to massless particles even for $\alpha = \pi$, and in the case $\alpha = \pi$ we verify that the massive states created by (5.15) and (5.18) have degeneracy two, and are hence organized into hypermultiplets.

The twist field (5.18) has a conformal weight

$$h\left(\sigma'_\alpha\right) = h\left(\sigma_\alpha\right) + \frac{1}{2}\left(1 - \frac{\alpha}{\pi}\right) = \frac{k}{4}\left(\frac{\alpha}{2\pi}\right)^2 + \frac{1}{2}\left(1 - \frac{\alpha}{\pi}\right) \tag{B.1}$$

and a $J^3_{\rm total}$ charge (recall (3.9) and (5.17))

$$m\left(\sigma'_\alpha\right) = m\left(\sigma_\alpha\right) + 1 = -\frac{k}{2}\frac{\alpha}{2\pi} + 1\,. \tag{B.2}$$

Its decomposition on $\mathrm{SU}(2)/\,\mathrm{U}(1) \times \mathrm{U}(1)$ thus reads

$$\sigma'_\alpha = z_1^{k-2} e^{-i\sqrt{\frac{2}{k}}(\frac{k}{2}\frac{\alpha}{2\pi}-1)Y}\,, \tag{B.3}$$

where z_1^{k-2} (the notation will become clear soon) is an operator in the $N = 2$ minimal model $\mathrm{SU}(2)_k/\,\mathrm{U}(1)$ with

$$h\left(z_1^{k-2}\right) = h\left(\sigma'_\alpha\right) - \frac{2}{k}\frac{\left(\frac{k}{2}\frac{\alpha}{2\pi} - 1\right)^2}{2} = \frac{1}{2}\frac{k-2}{k}\,. \tag{B.4}$$

It can be shown that z_1^{k-2} is a chiral operator in the $N = 2$ minimal model with a $\mathrm{U}(1)_R$ charge $R(z_1^{k-2}) = (k-2)/k$; it is the highest charge operator in the chiral ring of $\mathrm{SU}(2)_k/\,\mathrm{U}(1)$, $\{z_1^i | i = 0, 1, \ldots, k-2\}$, $R(z_1^i) = i/k$.

Collecting the above, recalling (5.19) (with σ_α replaced by σ'_α) and (5.21), (5.22) and (5.23), we find that the $4 - 6'$ vertex operator under consideration, and what it becomes in the cigar geometry (3.10), is

$$V'_{4-6'} = e^{-\varphi}\sigma_{45} S_{45} e^{ik_\mu x^\mu} e^{\beta\phi}\sigma'_\alpha \longrightarrow e^{-\varphi}\sigma_{45} S_{45} e^{ik_\mu x^\mu} z_1^{k-2} V_{j,-\frac{k}{2}\frac{\alpha}{2\pi}+1}\,. \tag{B.5}$$

The lowest lying state corresponds to $n = 1$ in (3.15), namely, $j = |m| - 1$; its mass is

$$\left(M'_\alpha\right)^2 = \frac{1}{2} + \frac{2}{k}j = \frac{1}{2} - \frac{2}{k} + \left|\frac{\alpha}{2\pi} - \frac{2}{k}\right|. \tag{B.6}$$

Since $j > -1/2$ (3.14), we have

$$\left(M'_\alpha\right)^2 > \frac{1}{2} - \frac{1}{k} \geq 0 \tag{B.7}$$

for any α allowed in the unitarity range.

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A particular case is the supersymmetric $\alpha = \pi$ configuration. In this case, the twist operators (5.15) and (5.18) take the form

$$\begin{aligned}\sigma_\pi^- &= e^{-\frac{i}{2}H} e^{-\frac{i}{2}\sqrt{\frac{k_B}{2}}u} = e^{-\frac{i}{2}\sqrt{\frac{k}{2}}Y}\,,\\ \sigma_\pi^+ &= e^{\frac{i}{2}H} e^{-\frac{i}{2}\sqrt{\frac{k_B}{2}}u} = z_1^{k-2} e^{-i\sqrt{\frac{2}{k}}(\frac{k}{4}-1)Y}\,.\end{aligned} \tag{B.8}$$

The corresponding $4-6'$ vertex operators turn in the cigar geometry (3.10) into

$$\begin{aligned}V^-_{4-6'} &= e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}e^{\beta\phi}\sigma_\pi^- \longrightarrow e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}V_{j,m_-}\,,\\ V^+_{4-6'} &= e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}e^{\beta\phi}\sigma_\pi^+ \longrightarrow e^{-\varphi}\sigma_{45}S_{45}e^{ik_\mu x^\mu}z_1^{k-2}V_{j,m_+}\,,\end{aligned} \tag{B.9}$$

where

$$m_\pm = -\frac{k-2}{4} \pm \frac{1}{2}\,. \tag{B.10}$$

Physical states in spacetime are obtained when j in $V^\pm_{4-6'}$ is (3.15)

$$j^n_\pm = |m_\pm| - n\,, \quad n = 1, 2, \ldots, \tag{B.11}$$

respectively. Notice that[16]

$$j^n_+ = j^{n+1}_-\,. \tag{B.12}$$

The on-shell conditions $h(V^\pm_{4-6'}) = 1$ imply

$$\frac{\left(M^n_\pm\right)^2}{2} = \frac{k-4}{16} - \frac{j^n_\pm\left(j^n_\pm+1\right)}{k}\,. \tag{B.13}$$

Using (B.10), (B.11), (B.12) and (B.13), one finds

$$\left(M^n_-\right)^2 = 7\frac{1}{k}(n-1)(k-2n)\,, \tag{B.14}$$

$$\left(M^n_+\right)^2 = \left(M^{n+1}_-\right)^2 = \frac{1}{k}n(k-2-2n)\,. \tag{B.15}$$

The unitarity bound (3.14) restricts n to a certain range. In this range $(M^n_+)^2 > 0$ for any n, while $(M^n_-)^2 \geq 0$ for any n and equality is satisfied iff $n = 1$. We thus see (B.15) that there is a degeneracy two for all massive states; they get organized into $4d$ hypermultiplets $(Q, \tilde{Q})$, as they should. The only massless state $M^1_- = 0$ (B.14) is non degenerate; it corresponds to the chiral superfield Q considered in section 5.1.

C. Multiplicity of 4-4 open strings

In this appendix we show that, except for (5.35), the other operators in (5.36) do not create massless states from the vacuum.

[16]Below we restrict to the case $k \geq 4$; for $k = 2, 3$ there are no massive excitations in the range (3.14).

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The operators in (5.36) are linear combinations of

$$V_m^a = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} \chi^a \sigma_{k_B/2,m} \,, \quad a = \pm, 3 \tag{C.1}$$

and

$$V_m^M = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} \psi^M \sigma_{k_B/2,m} \,, \quad M = \mu, 4, 5, r \,. \tag{C.2}$$

The operators $\sigma_{k_B/2,m}$ in (C.1) and (C.2) carry m units of charge under J^3 and have scaling dimension

$$h\left(\sigma_{k_B/2,m}\right) = \frac{1}{k}\frac{k_B}{2}\left(\frac{k_B}{2}+1\right) = \frac{k-2}{4} \,. \tag{C.3}$$

The decomposition of σ_{jm} on $\mathrm{SU}(2)/\mathrm{U}(1) \times \mathrm{U}(1)$ reads

$$\sigma_{jm} = V'_{jm} e^{i\sqrt{2/k}mY} \,, \tag{C.4}$$

where V'_{jm} is an operator in the $\mathrm{SU}(2)/\mathrm{U}(1)$ SCFT with scaling dimension

$$h\left(V'_{jm}\right) = \frac{j(j+1)-m^2}{k} \,. \tag{C.5}$$

Now, going from the cylinder (ϕ, Y) to the $\mathrm{SL}(2)/\mathrm{U}(1)$ cigar we take

$$e^{\beta\phi} e^{i\sqrt{2/k}mY} \longrightarrow V_{jm} \tag{C.6}$$

(see (5.21) and (5.22)). Altogether, in the background (3.10), the operators (C.2) take the form

$$V_m^M = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} \psi^M \sigma_{k_B/2,m} \longrightarrow e^{-\varphi} e^{ik_\mu x^\mu} \psi^M V'_{k_B/2,m} V_{jm} \,. \tag{C.7}$$

When $k_\mu^2 = 0$, the on-shell condition for V_m^M reads

$$\frac{1}{k}\frac{k_B}{2}\left(\frac{k_B}{2}+1\right) - \frac{j(j+1)}{k} = 0 \Longrightarrow j = \frac{k_B}{2} \,. \tag{C.8}$$

Since $|m| \leq k_B/2$, the condition (3.15) is not satisfied. Hence, no massless particle is emitted by V_m^M. Similarly, V_m^3 in (C.1) does not create massless states from the vacuum.

The operators $V_m^\pm$ in (C.1) include a factor $\chi^\pm \sigma_{k_B/2,m}$ which decomposes as

$$\chi^\pm \sigma_{k_B/2,m} = \Psi_m e^{i\sqrt{2/k}(m\pm 1)Y} \,, \tag{C.9}$$

where Ψ_m is in the $\mathrm{SU}(2)/\mathrm{U}(1)$ SCFT and has

$$h\left(\Psi_m\right) = \frac{k}{4} - \frac{(m\pm 1)^2}{k} \,. \tag{C.10}$$

In the geometry (3.10), the operators $V_m^\pm$ now take the form

$$V_m^\pm = e^{-\varphi} e^{ik_\mu x^\mu} e^{\beta\phi} \chi^\pm \sigma_{k_B/2,m} \longrightarrow e^{-\varphi} e^{ik_\mu x^\mu} \Psi_m V_{j,m\pm 1} \,. \tag{C.11}$$

When $k_\mu^2 = 0$, the on-shell condition gives again $j = k_B/2$ (C.8). Since $m \pm 1$ take values between $-k_B/2 \pm 1$ and $k_B/2 \pm 1$, only the operators $V^+_{k_B/2}$ and $V^-_{-k_B/2}$ — the operators in (5.36) whose SU(2) part is (5.35) — satisfy (3.15). Therefore, the other operators $V_m^\pm$ in (C.1) do not emit massless particles.

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References

[1] A. Giveon and D. Kutasov, *Brane dynamics and gauge theory*, *Rev. Mod. Phys.* **71** (1999) 983 [hep-th/9802067].

[2] For a review see O. Aharony, *A brief review of 'little string theories'*, *Class. and Quant. Grav.* **17** (2000) 929 [hep-th/9911147].

[3] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, NS5-branes and holography*, *J. High Energy Phys.* **10** (1998) 004 [hep-th/9808149].

[4] C.G. Callan Jr., J.A. Harvey and A. Strominger, *Supersymmetric string solitons*, hep-th/9112030.

[5] S. Elitzur, A. Giveon and D. Kutasov, *Branes and $N = 1$ duality in string theory*, *Phys. Lett.* **B 400** (1997) 269 [hep-th/9702014];
S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and A. Schwimmer, *Brane dynamics and $N = 1$ supersymmetric gauge theory*, *Nucl. Phys.* **B 505** (1997) 202 [hep-th/9704104].

[6] A. Hanany and E. Witten, *Type-IIB superstrings, BPS monopoles and three-dimensional gauge dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [hep-th/9611230].

[7] J.H. Brodie and A. Hanany, *Type-IIA superstrings, chiral symmetry and $N = 1$ 4d gauge theory dualities*, *Nucl. Phys.* **B 506** (1997) 157 [hep-th/9704043].

[8] A. Hanany and K. Hori, *Branes and $N = 2$ theories in two dimensions*, *Nucl. Phys.* **B 513** (1998) 119 [hep-th/9707192].

[9] K. Landsteiner, E. Lopez and D.A. Lowe, *Duality of chiral $N = 1$ supersymmetric gauge theories via branes*, *J. High Energy Phys.* **02** (1998) 007 [hep-th/9801002].

[10] S. Elitzur, A. Giveon, D. Kutasov and D. Tsabar, *Branes, orientifolds and chiral gauge theories*, *Nucl. Phys.* **B 524** (1998) 251 [hep-th/9801020].

[11] I. Brunner, A. Hanany, A. Karch and D. Lust, *Brane dynamics and chiral non-chiral transitions*, *Nucl. Phys.* **B 528** (1998) 197 [hep-th/9801017].

[12] A. Giveon, D. Kutasov and O. Pelc, *Holography for non-critical superstrings*, *J. High Energy Phys.* **10** (1999) 035 [hep-th/9907178].

[13] A. Giveon and D. Kutasov, *Little string theory in a double scaling limit*, *J. High Energy Phys.* **10** (1999) 034 [hep-th/9909110];
Comments on double scaled little string theory, *J. High Energy Phys.* **01** (2000) 023 [hep-th/9911039].

[14] H. Ooguri and C. Vafa, *Two-dimensional black hole and singularities of CY manifolds*, *Nucl. Phys.* **B 463** (1996) 55 [hep-th/9511164].

[15] K. Sfetsos, *Branes for Higgs phases and exact conformal field theories*, *J. High Energy Phys.* **01** (1999) 015 [hep-th/9811167];
Rotating NS5-brane solution and its exact string theoretical description, *Fortsch. Phys.* **48** (2000) 199 [hep-th/9903201].

JHEP08(2000)046

[16] S. Elitzur, A. Forge and E. Rabinovici, *Some global aspects of string compactifications*, *Nucl. Phys.* **B 359** (1991) 581;
G. Mandal, A.M. Sengupta and S.R. Wadia, *Classical solutions of two-dimensional string theory*, *Mod. Phys. Lett.* **A 6** (1991) 1685;
E. Witten, *On string theory and black holes*, *Phys. Rev.* **D 44** (1991) 314.

[17] J. Teschner, *On structure constants and fusion rules in the* $\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2)$ *WZNW model*, *Nucl. Phys.* **B 546** (1999) 390 [hep-th/9712256];
J. Teschner, *The mini-superspace limit of the* $\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2)$ *WZNW model*, *Nucl. Phys.* **B 546** (1999) 369 [hep-th/9712258];
Operator product expansion and factorization in the h-3+ WZNW model, *Nucl. Phys.* **B 571** (2000) 555 [hep-th/9906215];
V.A. Fateev, A.B. Zamolodchikov and Al.B. Zamolodchikov, unpublished.

[18] N. Seiberg, *Notes on quantum Liouville theory and quantum gravity*, *Prog. Theor. Phys. Suppl.* **102** (1990) 319;
J. Polchinski, *Strings '90*, College Station Workshop, 1990.

[19] A. Giveon and D. Kutasov, unpublished.

[20] J. Polchinski, S. Chaudhuri and C.V. Johnson, *Notes on D-branes*, hep-th/9602052.

[21] A. Hashimoto, *Dynamics of Dirichlet-Neumann open strings on D-branes*, *Nucl. Phys.* **B 496** (1997) 243 [hep-th/9608127].

[22] P. Di Francesco, P. Mathieu and D. Senechal, *Conformal field theory*, Springer, New York1997.

[23] G. Pradisi, A. Sagnotti and Y.S. Stanev, *Planar duality in* SU(2) *WZW models*, *Phys. Lett.* **B 354** (1995) 279 [hep-th/9503207];
The open descendants of nondiagonal SU(2) *WZW models*, *Phys. Lett.* **B 356** (1995) 230 [hep-th/9506014];
Completeness conditions for boundary operators in 2d conformal field theory, *Phys. Lett.* **B 381** (1996) 97 [hep-th/9603097].

[24] C. Klimcik and P. Severa, *Open strings and D-branes in WZNW models*, *Nucl. Phys.* **B 488** (1997) 653 [hep-th/9609112].

[25] M. Kato and T. Okada, *D-branes on group manifolds*, *Nucl. Phys.* **B 499** (1997) 583 [hep-th/9612148].

[26] S. Forste, D. Ghoshal and S. Panda, *An orientifold of the solitonic fivebrane*, *Phys. Lett.* **B 411** (1997) 46 [hep-th/9706057].

[27] M. Bianchi and Y.S. Stanev, *Open strings on the Neveu-Schwarz pentabrane*, *Nucl. Phys.* **B 523** (1998) 193 [hep-th/9711069].

[28] A.Y. Alekseev and V. Schomerus, *D-branes in the WZW model*, *Phys. Rev.* **D 60** (1999) 061901 [hep-th/9812193].

JHEP08(2000)046

[29] S. Stanciu, *D-branes in an AdS_3 background*, *J. High Energy Phys.* **09** (1999) 028 [hep-th/9901122].

[30] K. Gawedzki, *Conformal field theory: a case study*, hep-th/9904145.

[31] L. Birke, J. Fuchs and C. Schweigert, *Symmetry breaking boundary conditions and WZW orbifolds*, hep-th/9905038.

[32] H. Garcia-Compean and J.F. Plebanski, *D-branes on group manifolds and deformation quantization*, hep-th/9907183.

[33] R.E. Behrend, P.A. Pearce, V.B. Petkova and J.-B. Zuber, *Boundary conditions in rational conformal field theories*, *Nucl. Phys.* **B 570** (2000) 525 [hep-th/9908036].

[34] A.Y. Alekseev, A. Recknagel and V. Schomerus, *Non-commutative world-volume geometries: branes on* SU(2) *and fuzzy spheres*, *J. High Energy Phys.* **09** (1999) 023 [hep-th/9908040].

[35] G. Felder, J. Frohlich, J. Fuchs and C. Schweigert, *The geometry of WZW branes*, *J. Geom. Phys.* **34** (2000) 162 [hep-th/9909030].

[36] See, for example S. Stanciu, *D-branes in group manifolds*, *J. High Energy Phys.* **01** (2000) 025 [hep-th/9909163].

[37] J.M. Figueroa-O'Farrill and S. Stanciu, *D-branes in $AdS_3 \times S_3 \times S_3 \times S_1$*, *J. High Energy Phys.* **04** (2000) 005 [hep-th/0001199].

[38] C. Bachas, M. Douglas and C. Schweigert, *Flux stabilization of D-branes*, *J. High Energy Phys.* **05** (2000) 048 [hep-th/0003037].

[39] A.Y. Alekseev, A. Recknagel and V. Schomerus, *Brane dynamics in background fluxes and non-commutative geometry*, *J. High Energy Phys.* **05** (2000) 010 [hep-th/0003187].

[40] J.L. Cardy, *Boundary conditions, fusion rules and the Verlinde formula*, *Nucl. Phys.* **B 324** (1989) 581.

[41] I. Antoniadis, S. Ferrara and C. Kounnas, *Exact supersymmetric string solutions in curved gravitational backgrounds*, *Nucl. Phys.* **B 421** (1994) 343 [hep-th/9402073].

[42] T. Eguchi and Y. Sugawara, *Modular invariance in superstring on Calabi-Yau N-fold with ADE singularity*, *Nucl. Phys.* **B 577** (2000) 3 [hep-th/0002100].

[43] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[44] S. Mukhi, N.V. Suryanarayana and D. Tong, *Brane-antibrane constructions*, *J. High Energy Phys.* **03** (2000) 015 [hep-th/0001066].

[45] S. Mukhi and N.V. Suryanarayana, *A stable non-BPS configuration from intersecting branes and antibranes*, *J. High Energy Phys.* **06** (2000) 001 [hep-th/0003219].

[46] Y. Yoshida, *Geometrical analysis of brane creation via M-theory*, *Prog. Theor. Phys.* **99** (1998) 305 [hep-th/9711177].

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